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EXISTENCE AND NONEXISTENCE OF GLOBAL SOLUTION OF CAUCHY PROBLEM FOR A CLASS OF SYSTEM OF SEMI-LINEAR HYPERBOLIC EQUATIONS OF FOURTH ORDER WITH DAMPING

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Abstract. The Cauchy problem for a class of system of semi-linear hyperbolic equations containing partial derivatives of fourth order with respect to some variables and second order partial derivatives with respect to some other variables is investigated. A theorem of non existence and existence of global solution is proved.

1. Introduction

The existence of global solution to the Cauchy problem for semi-linear hyperbolic equations with damping was investigated in [8, 11, 15, 18, 19, 23].

The global solvability of the Cauchy problem for hyperbolic equations of higher order and for the system of semi-linear hyperbolic equations was studied in [1-3, 5, 7, 13, 14, 20] and [4, 6, 9, 16, 21, 22, 24] respectively.

In this work, we study the Cauchy problem for a system of semilinear hyperbolic equations with anisotropic elliptic parts, where every equation includes second and fourth order derivatives in various spatial variables.

We consider the following initial-boundary value problem:

$$\left.\begin{aligned} u_{1tt}+u_{1t}+\Delta_{I_1}^2 u_1-\Delta_{J_1}u_1=\sum_{k=1}^{l_1} f_{1k}(u_1,u_2)\\ u_{2tt}+u_{2t}+\Delta_{I_2}^2 u_2-\Delta_{J_2}u_2=\sum_{k=1}^{l_2} f_{2k}(u_1,u_2)\end{aligned}\right\}, \tag{1.1}$$

$$u_i\left(0,x\right)=\varphi_i\left(x\right),\ \ u_{it}\left(0,x\right)=\psi_i\left(x\right),\ \ x\in R_N,\ \ i=1,2, \tag{1.2}$$

where $\Delta_{I_i}=\sum\limits_{s\in I_i}\dfrac{\partial^2}{\partial x_s^2}$, $\Delta_{J_i}=\sum\limits_{s\in J_i}\dfrac{\partial^2}{\partial x_s^2}$, $I_i\subset N_n=\{1,...,n\}$, $J_i=N_n\backslash I_i$, $i=1,2$ and $f_{ik}:R^2\to R$, $k=1,2,...,l_i$, $i=1,2$ are given functions which will be specified later.

Problems of this type arise in material science and physics (see.[12]).

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Let denote by $m_r = \overline{\overline{J_r}}, r = 1, 2$ the number of the elements of J_r, by $n_r = \overline{\overline{I_r}} = n - m_r$ the number of the elements of I_r. For definiteness we suppose that

$$m_1 \geq m_2. \tag{1.3}$$

Our interest is focused on a critical exponent which classifies the global existence and the finite time blow up of the solution for small data.

For the Cauchy problem:

$$\left.\begin{array}{l} u_{tt} + u_t - \Delta u = |u|^p, t > 0, \ \ x \in R^n \\ \\ u(0, x) = u_0(x), \ \ u_t(0, x) = u_1(x), \ x \in R^n \end{array}\right\} \tag{1.4}$$

the critical exponent is known as $p_c = 1 + \dfrac{2}{n}$ and called the Fujita exponent (see [7, 10]). More precisely, if $1 < p \leq 1 + \dfrac{2}{n}$, then the solution of the problem (1.4) for the small data blow up in a finite time, and for $p > 1 + \dfrac{2}{n}$, the small solution exists globally in time.

We assume that

$$n + m_1 \leq 4, \tag{1.5}$$

and the functions $f_{ik}, k = 1, 2, ..., l_i, \ i = 1, 2$ satisfy the following conditions

1) $f_{ik}(.), k = 1, 2, ..., l_i, \ i = 1, 2$ are continuously differentiable functions on R^2;

2) For any $(u_1, u_2) \in R^2$ the following estimation is satisfied.

$$|f_{ik}(u_1, u_2)| \leq C_{ik} |u_1|^{p_{ik}} |u_2|^{q_{ii}} \ , k = 1, 2, ..., l_i, \quad i = 1, 2,$$

where

$$p_{ik} \geq 0, \ \ q_{ik} \geq 0, \ \ p_{ik} + q_{ii} \geq 2, \ \ k = 1, 2, ..., l_i, \quad i = 1, 2, \tag{1.6}$$

$$\frac{n + m_1}{4} p_{ik} + \frac{n + m_2}{4} q_{ik} > 1 + \Psi(q_{ik}). \tag{1.7}$$

Here

$$\Psi(s) = \begin{cases} \dfrac{m_1 - m_2}{4}, & s \geq 2, \\ \\ \dfrac{(m_1 - m_2)(2 - s)}{8}, & 0 \leq s < 2 \ . \end{cases}$$

In future we use the following abbreviations: $\|\,.\,\|_2$ and $\|\,.\,\|_p$, which denote, the usual norms in $L_2(R^n)$ and $L_p(R^n)$ respectively. This paper is organized as follows. The global existence and the decay of the solution are given in section 2. In section 3, we show the blow up properties of solution in case $l_1 = l_2 = 1$ and $f_{11} = f_1(u_2), f_{21} = f_2(u_1)$.

2. The existence of global solutions

We denote by $W_{2,k}^{2s,s} = W_2^{2s,s}(I_k, J_k), \ k = 1, 2, \ s = 0, 1, 2, ...$ the functional spaces with the norm:

$$\|u\|^2_{W_{2,k}^{2s,s}} = \left\{ \|u\|^2_{L_2(R_N)} + \sum_{i \in I_k} \left\|D_{x_i}^{2s} u\right\|^2 + \sum_{j \in J_k} \left\|D_{x_j}^{s} u\right\|^2 \right\}^{\frac{1}{2}}, \ k = 1, 2, \ s = 1, 2, ... \ .$$

Let U_δ^k be a ball of radius $\delta > 0$ centered at zero in the spaces

$$\left[W_{2,k}^{2,1} \cap L_1(R^n)\right] \times \left[L_2(R^n) \cap L_1(R^n)\right], i.e.$$

$$U_\delta^k = \left\{(u,v) : \| u \|_{W_{2,k}^{2,1}} + \| u \|_{L_1(R^n)} + \| v \|_{L_2(R^n)} + \| v \|_{L_1(R^n)} < \delta\right\}.$$

We denote by $C\left([0,T]; W_{2,1}^{2,1} \times W_{2,2}^{2,1}\right)$ the space of all continuous functions $t \to (u_1,u_2) : [0,T] \to W_{2,1}^{2,1} \times W_{2,2}^{2,1}$ with the norm

$$\|(u_1,u_2)\|_{C\left([0,T]; W_{2,1}^{2,1} \times W_{2,2}^{2,1}\right)} = \max_{0 \le t \le T}\left[\|u_1(t,.)\|_{W_{2,1}^{2,1}} + \|u_2(t,.)\|_{W_{2,2}^{2,1}}\right]$$

and by $C^1([0,T]; L_2(R^n) \times L_2(R^n))$ the space of all differentiable functions $t \to (u_1,u_2) : [0,T] \to L_2(R^n) \times L_2(R^n)$ with the norm

$$\|(u_1,u_2)\|_{C^1([0,T];L_2(R^n)\times L_2(R^n))} = \max_{0 \le t \le T} \sum_{i=1}^{2}\left[\|u_i(t,.)\|_{L_2(R^n)} + \left\|u'_{it}(t,.)\right\|_{2(R^n)}\right].$$

We use the notations: $C\left([0.\infty); W_{2,1}^{2,1} \times W_{2,2}^{2,1}\right) = \bigcup_{T>0} C\left([0,T]; W_{2,1}^{2,1} \times W_{2,2}^{2,1}\right)$ and $C^1([0,\infty); L_2(R^n) \times L_2(R^n)) = \bigcup_{T>0} C^1([0,T]; L_2(R^n) \times L_2(R^n))$.

The following theorem is proved using the method given in the papers [16, 24]

Theorem 2.1. *Let the supposition* (1.3) *and the conditions 1-3 are fulfilled. Then there exists $\delta > 0$ such that, for any $((\varphi_1,\psi_1),(\varphi_2,\psi_2)) \in U_\delta^1 \times U_\delta^2$ the problem* (1.1), (1.2) *has a unique solution*

$$(u_1,u_2) \in C\left([0,\infty); W_{2,1}^{2,1} \times W_{2,2}^{2,1}\right) \bigcap C^1([0,\infty); L_2(R_n) \times L_2(R_n))$$

and the following estimates are valid:

$$\|u_:(\cdot,t)\|_{L_2(R_n)} \le C(\delta)(1+t)^{-\frac{n+m_k}{8}};$$

$$\sum_{i \in I_k}\left\|D_{x_i}^2 u_k(\cdot,t)\right\|_{L_2(R_n)} + \sum_{j \in J_k}\left\|D_{x_j} u_k(\cdot,t)\right\|_{L_2(R_n)} \le C(\delta)(1+t)^{-\frac{n+m_k+4}{8}};$$

$$\|u_{k_t}(t,\cdot)\|_{L_2(R_n)} \le C(\delta)(1+t)^{-\gamma_k},$$

where

$$\gamma_k = \min\left\{\frac{n+m_k+8}{8}, \frac{(p_k-1)(n+m_k)}{8}\right\}, \quad k = 1,2.$$

In particularly for the Cauchy problem

$$\left.\begin{array}{l} u_{1tt} + u_{1t} + \Delta_{I_1}^2 u_1 - \Delta_{J_1} u_1 = f_1(u_2) \\ u_{2tt} + u_{2t} + \Delta_{I_2}^2 u_2 - \Delta_{J_2} u_2 = f_2(u_1) \end{array}\right\}, \tag{2.1}$$

$$u_i(0,x) = \varphi_i(x), \ u_{it}(0,x) = \psi_i(x), \ x \in R^n, \ i = 1,2, \tag{2.2}$$

where

$$|f_1(u_2)| \le C|u_2|^{p_2}, |f_2(u_1)| \le C|u_1|^{q_1}, \tag{2.3}$$

the condition (1.7) is written as follows

$$p_2 \ge 0, \quad q_1 \ge 0, \quad p_2 + q_1 \ge 2, \tag{2.4}$$

$$p_2 > 1 + \frac{4}{n+m_1}, \quad q_1 > 1 + \frac{4}{n+m_2}. \tag{2.5}$$

Therefore, under the conditions (2.3)- (2.5) for sufficiently small initial data, the problem (1.6), (1.7) has a global solution.

3. Non-existence of global solutions

Let us to discuss the condition (2.3), (2.5). That is, under the assumption

$$|f_1(u_2)| \geq A\,|u_2|^{p_2}\,, \quad |f_2(u_1)| \geq A\,|u_1|^{q_1}\,, \tag{3.1}$$

where

$$\max\left\{(n+m_2)\,p_2, (n+m_1)\,q_1+m_2-m_1\right\} \leq \min\left\{4p_2', 4q_1'+m_2-m_1\right\}, \tag{3.2}$$

we will derive the blow-up property to the Cauchy problem (2.1), (2.2).

Suppose that

$$\varphi_i\,(.)\ ,\ \psi_i(.) \in L_1\,(R^n)\,, \quad i=1,2, \tag{3.3}$$

The functions u_1, u_2 are called the weak solution of the Cauchy problem (2.1), (2.2) if

- $u_1\ \in L_{p_2,loc}\,([0,\infty)\times R^n)$, $u_2\ \in L_{q_1,loc}\,([0,\infty)\times R^n)$;
- $f_1(u_2)$, $f_i(u_1) \in L_{1,loc}\,([0,\infty)\times R^n)$;
- $u_1(t,x)$, $u_2(t,x)$ satisfy the following system of equalities:

$$-\int\limits_{R^n}(\varphi_1\,(x)+\psi_1\,(x))\,\xi_1\,(x,0)\,dx+\int\limits_{R^n}\varphi_1\,(x)\,\frac{\partial\xi_1\,(x,0)}{\partial t}dx+$$
$$+\int\limits_0^\infty\int\limits_{R^n}u_1\frac{\partial^2\xi_1}{\partial t}dxdt-\int\limits_0^\infty\int\limits_{R^n}u_1\frac{\partial\xi_1}{\partial t}dxdt-$$
$$-\int\limits_0^\infty\int\limits_{R^n}u_1\left[\Delta_{I_1}^2\xi_1-\Delta\xi_1\right]dxdt=\int\limits_0^\infty\int\limits_{R^n}f_1\,(u_2)\,\xi_1dxdt; \tag{3.4}$$

$$-\int\limits_{R^n}(\varphi_2\,(x)+\psi_2\,(x))\,\xi_2\,(x,0)\,dx+\int\limits_{R^n}\varphi_2\,(x)\,\frac{\partial\xi_2\,(x,0)}{\partial t}dx+$$
$$+\int\limits_0^\infty\int\limits_{R^n}u_2\frac{\partial^2\xi_2}{\partial t}dxdt-\int\limits_0^\infty\int\limits_{R^n}u_2\frac{\partial\xi_2}{\partial t}dxdt-$$
$$-\int\limits_0^\infty\int\limits_{R^n}u_2\left[\Delta_{I_2}^2\xi_1-\Delta\xi_2\right]dxdt=\int\limits_0^\infty\int\limits_{R^n}f_2\,(u_1)\,\xi_2dxdt \tag{3.5}$$

for any functions $\xi_1,\xi_2\in C_0^{2,2}\,([0,\infty)\times R^n)\,,\ \xi_i(t,x)\geq 0,\ i=1,2.$

Theorem 3.1. *Let conditions* (3.1)- (3.3) *be satisfied, and*

$$\sum_{i=1}^{2}\int\limits_{R^n}(\varphi_i\,(x)+\psi_i\,(x))\,dx\geq 0\,, \quad i=1,2. \tag{3.6}$$

Then the problem (2.1), (2.2) *has no nontrivial solutions.*

The proof of theorem 3.1. We will prove the theorem by the method of test functions (see [16, 24]). We choose the test functions as follows

$$\xi_i(x,t)=\varphi\left(\frac{t^{\varkappa}+\sum\limits_{k\in I_i}|x_k|^{\lambda_k}+\sum\limits_{k\in J_i}|x_k|^{\nu_k}}{d^2}\right),$$

where $\varkappa>0$, , $\mu_i>0$, $\nu_i>0$ and $d>0$ are some parameters that are defined below. Here $\varphi(\cdot)\in C_0^{\infty}(R)$ and $\varphi(r)=1$, if $0\le r\le 1$, $\varphi(r)=0$ if $r\ge 2$. From definition of $\xi_i(x,t)$, $i=1,2$ it follows that

$$\frac{\partial\xi_i(0,x)}{\partial t}=0,\ \ i=1,2. \tag{3.7}$$

Further, applying the Holder and Young's inequalities, from (3.1), (3.4), (3.5) and (3.7) we get

$$\begin{aligned}
&\int\limits_{R^n}(\varphi_1(x)+\psi_1(x))\,\xi_1(x,0)\,dx+\int\limits_0^{\infty}\int\limits_{R^n}|u_2|^{q_1}\xi_1dxdt\le\\
&\le 4\varepsilon\int\limits_0^{\infty}\int\limits_{R^n}|u_1|^{p_2}\xi_2dxdt+\left[\frac{1}{\varepsilon}\int\limits_0^{\infty}\int\limits_{R^n}\xi_2^{-\frac{p_2'}{p_2}\rho'}\right]^{1/\rho'}\times\\
&\times\left\{\left[\int\limits_0^{\infty}\int\limits_{R^n}\left|\frac{\partial^2\xi_1}{\partial t^2}\right|^{p_2'\rho}dxdt\right]^{1/\rho}+\left[\int\limits_0^{\infty}\int\limits_{R^n}\left|\frac{\partial\xi_1}{\partial t}\right|^{p_2'\rho}dxdt\right]^{1/\rho}+\right.\\
&\left.+\left[\int\limits_0^{\infty}\int\limits_{R^n}\left|\Delta_{I_1}^2\xi_1\right|^{p_2'\rho}dxdt\right]^{1/\rho}+\left[\int\limits_0^{\infty}\int\limits_{R^n}\left|\Delta_{J_1}\xi_1\right|^{p_2'\rho}\right]^{1/\rho}dxdt\right\};
\end{aligned}\tag{3.8}$$

$$\begin{aligned}
&\int\limits_{R^n}(\varphi_2(x)+\psi_2(x))\,\xi_2(x,0)\,dx+\int\limits_0^{\infty}\int\limits_{R^n}|u_1|^{p_2}\xi_2dxdt\le\\
&\le 4\varepsilon\int\limits_0^{\infty}\int\limits_{R^n}|u_2|^{q_1}\xi_1dxdt+\left[\frac{1}{\varepsilon}\int\limits_0^{\infty}\int\limits_{R^n}\xi_1^{-\frac{q_1'}{q_1}\rho'}\right]^{1/\rho'}\times\\
&\times\left\{\left[\int\limits_0^{\infty}\int\limits_{R^n}\left|\frac{\partial^2\xi_2}{\partial t^2}\right|^{q_1'\rho}dxdt\right]^{1/\rho}+\left[\int\limits_0^{\infty}\int\limits_{R^n}\left|\frac{\partial\xi_2}{\partial t}\right|^{q_1'\rho}dxdt\right]^{1/\rho}+\right.\\
&\left.+\left[\int\limits_0^{\infty}\int\limits_{R^n}\left|\Delta_{I_2}^2\xi_2\right|^{q_1\rho}dxdt\right]^{1/\rho}+\left[\int\limits_0^{\infty}\int\limits_{R^n}\left|\Delta_{J_2}\xi_2\right|^{q_1'\rho}\right]^{1/\rho}dxdt\right\},
\end{aligned}\tag{3.9}$$

where $\rho>1,\rho'=\dfrac{\rho}{\rho-1}$.

We substitute the variable in the right side of inequalities (3.8), (3.9):

$$t=d^{2/\chi},\ \ x_k=d^{2/\lambda_k}y_k,\ \ k\in I_i,\ \ x_k=d^{2/\nu_k}y_k,\ \ k\in J_i,\ \ i=1,2.$$

By denoting $\Omega_i = \left\{ (y,\tau) \in R^N \times R_+,\ \ \tau^\chi + \sum\limits_{k\in I_i} |y_k|^{\lambda_k} + \sum\limits_{k\in J_i} |y_k|^{\nu_k} \le 2 \right\}$, and

$$h_i(y,\tau) = \tau^\chi + \sum_{k\in I_i} |y_k|^{\lambda_k} + \sum_{k\in J_i} |y_k|^{\nu_k},$$

we have

$$\left[\int\limits_0^\infty \int\limits_{R^n} \xi_2^{-\frac{p_2'}{p_2}\rho'} dxdt\right]^{1/\rho'} = d^{\frac{1}{\rho'}\left[\frac{2}{\varkappa} + \sum\limits_{k\in I_2} \frac{1}{\lambda_k} + \sum\limits_{k\in J_2} \frac{1}{\nu_k}\right]} \left[\iint\limits_{\Omega_2} |\varphi\circ h_2|^{-\frac{p_2'}{p_2}\rho''} dyd\tau\right]^{1/\rho''};$$

$$\left[\int\limits_0^\infty \int\limits_{R^n} \left|\frac{\partial \xi_1}{\partial t}\right|^{p_2'\rho} dxdt\right]^{1/\rho} =$$

$$= d^{-\frac{2p_2'}{\chi} + \frac{1}{\rho}\left[\frac{2}{\varkappa} + \sum\limits_{k\in I_2} \frac{1}{\lambda_k} + \sum\limits_{k\in J_2} \frac{1}{\nu_k}\right]} \left[\iint\limits_{\Omega_1} \left|\frac{\partial}{\partial\tau}(\varphi\circ h_1)\right|^{p_2'\rho} dyd\tau\right]^{1/\rho};$$

$$\left[\int\limits_0^\infty \int\limits_{R^n} \left|\frac{\partial^2 \xi_1}{\partial t^2}\right|^{p_2'\rho} dxdt\right]^{1/\rho} =$$

$$= d^{-\frac{4p_2'}{\chi} + \frac{1}{\rho}\left[\frac{2}{\varkappa} + \sum\limits_{k\in I_2} \frac{1}{\lambda_k} + \sum\limits_{k\in J_2} \frac{1}{\nu_k}\right]} \left[\iint\limits_{\Omega_1} \left|\frac{\partial^2}{\partial\tau^2}(\varphi\circ h_1)\right|^{p_2'\rho} dyd\tau\right]^{1/\rho};$$

if $k \in I_1$:

$$\left[\int\limits_0^\infty \int\limits_{R^n} \left|\frac{\partial^4 \xi_1}{\partial x_k^4}\right|^{p_2'\rho} dxdt\right]^{1/\rho} =$$

$$= d^{-\frac{4p_2'}{\lambda_k} + \frac{1}{\rho}\left[\frac{2}{\varkappa} + \sum\limits_{k\in I_2} \frac{1}{\lambda_k} + \sum\limits_{k\in J_2} \frac{1}{\nu_k}\right]} \left[\iint\limits_{\Omega_1} \left|\frac{\partial^4}{\partial y_k^4}(\varphi\circ h_1)\right|^{p_2'\rho} dyd\tau\right]^{1/\rho};$$

if $k \in J_1$:

$$\left[\int\limits_0^\infty \int\limits_{R^n} \xi_1^{-\frac{q_1'}{q_1}\rho'} dxdt\right]^{1/\rho'} = d^{\frac{1}{\rho'}\left[\frac{2}{\varkappa} + \sum\limits_{k\in I_1} \frac{1}{\lambda_k} + \sum\limits_{k\in J_1} \frac{1}{\nu_k}\right]} \left[\iint\limits_{\Omega_2} |\varphi\circ h_1|^{-\frac{q_1}{q_1}\rho''} dyd\tau\right]^{1/\rho''};$$

$$\left[\int\limits_0^\infty \int\limits_{R^n} \left|\frac{\partial \xi_2}{\partial t}\right|^{q_1'\rho} dxdt\right]^{1/\rho} =$$

$$= d^{-\frac{2q_1'}{\chi} + \frac{1}{\rho}\left[\frac{2}{\varkappa} + \sum\limits_{k\in I_1} \frac{1}{\lambda_k} + \sum\limits_{k\in J_1} \frac{1}{\nu_k}\right]} \left[\iint\limits_{\Omega_2} \left|\frac{\partial}{\partial\tau}(\varphi\circ h_2)\right|^{q_1'\rho} dyd\tau\right]^{1/\rho};$$

$$\left[\int_0^{\infty}\int_{R^n}\left|\frac{\partial^2\xi_2}{\partial t^2}\right|^{q_1'\rho}dxdt\right]^{1/\rho}=$$

$$=d^{-\frac{4q_1'}{\chi}+\frac{1}{\rho}\left[\frac{2}{\varkappa}+\sum\limits_{k\in I_1}\frac{1}{\lambda_k}+\sum\limits_{k\in J_1}\frac{1}{\nu_k}\right]}\left[\iint_{\Omega_2}\left|\frac{\partial^2}{\partial\tau^2}(\varphi\circ h_2)\right|^{q_1'\rho}dyd\tau\right]^{1/\rho};$$

if $k\in I_2$:

$$\left[\int_0^{\infty}\int_{R^n}\left|\frac{\partial^4\xi_2}{\partial x_k^4}\right|^{q_1'\rho}dxdt\right]^{1/\rho}=$$

$$=d^{-\frac{4q_1'}{\lambda_k}+\frac{1}{\rho}\left[\frac{2}{\varkappa}+\sum\limits_{k\in I_1}\frac{1}{\lambda_k}+\sum\limits_{k\in J_1}\frac{1}{\nu_k}\right]}\left[\iint_{\Omega_2}\left|\frac{\partial^4}{\partial y_k^4}(\varphi\circ h_2)\right|^{q_1'\rho}dyd\tau\right]^{1/\rho};$$

if $k\in J_2$:

$$\left[\int_0^{\infty}\int_{R^n}\left|\frac{\partial^2\xi_2}{\partial x_k^2}\right|^{q_1'\rho}dxdt\right]^{1/\rho}=$$

$$=d^{-\frac{2q_1'}{\lambda_k}+\frac{1}{\rho}\left[\frac{2}{\varkappa}+\sum\limits_{k\in I_1}\frac{1}{\lambda_k}+\sum\limits_{k\in J_1}\frac{1}{\nu_k}\right]}\left[\iint_{\Omega_2}\left|\frac{\partial^2}{\partial y_k^2}(\varphi\circ h_2)\right|^{q_1'\rho}dyd\tau\right]^{1/\rho}.$$

Since on the right hand sides of all the above equalities the integrand functions are continuous, they are bounded. Using this, from (3.8), (3.9) we obtain that

$$\sum_{j=1}^{2}\int_{R^n}\left(\varphi_j\left(x\right)+\psi_j\left(x\right)\right)\xi_j(x,0)dx+$$

$$+(1-4\varepsilon)\Big[\iint\limits_{t^{\varkappa}+\sum\limits_{k\in I_i}|x_k|^{\lambda_k}+\sum\limits_{k\in J_i}|x_k|^{\nu_k}\le d^2}|u_2|^{q_1}\,dxdt+$$

$$+\iint\limits_{t^{\varkappa}+\sum\limits_{k\in I_2}|x_k|^{\lambda_k}+\sum\limits_{k\in J_2}|x_k|^{\nu_k}\le d^2}|u_1|^{p_2}\,dxdt\Big]\le$$

$$\le C\left\{d^{-\frac{2p_2'}{\chi}+\left[\frac{2}{\varkappa}+\sum\limits_{k\in I_2}\frac{1}{\lambda_k}+\sum\limits_{k\in J_2}\frac{1}{\nu_k}\right]}+d^{-\frac{4p_2'}{\chi}+\left[\frac{2}{\varkappa}+\sum\limits_{k\in I_2}\frac{1}{\lambda_k}+\sum\limits_{k\in J_2}\frac{1}{\nu_k}\right]}+\right.$$

$$+\sum_{j\in I_1} d^{-\frac{4p_2'}{\lambda_j}+\left[\frac{2}{\varkappa}+\sum\limits_{k\in I_2}\frac{1}{\lambda_k}+\sum\limits_{k\in J_2}\frac{1}{\nu_k}\right]}+\sum_{j\in J_1} d^{-\frac{4p_2'}{\nu_j}+\left[\frac{2}{\varkappa}+\sum\limits_{k\in I_2}\frac{1}{\lambda_k}+\sum\limits_{k\in J_2}\frac{1}{\nu_k}\right]}+$$
$$+d^{-\frac{2q_1'}{\chi}+\left[\frac{2}{\varkappa}+\sum\limits_{k\in I_1}\frac{1}{\lambda_k}+\sum\limits_{k\in J_1}\frac{1}{\nu_k}\right]}+d^{-\frac{4q_1''}{\chi}+\left[\frac{2}{\varkappa}+\sum\limits_{k\in I_1}\frac{1}{\lambda_k}+\sum\limits_{k\in J_1}\frac{1}{\nu_k}\right]}+$$
$$\left.+\sum_{j\in I_2} d^{-\frac{4q_1'}{\lambda_j}+\left[\frac{2}{\varkappa}+\sum\limits_{k\in I_1}\frac{1}{\lambda_k}+\sum\limits_{k\in J_1}\frac{1}{\nu_k}\right]}+\sum_{j\in I_2} d^{-\frac{4q_1'}{\nu_j}+\left[\frac{2}{\varkappa}+\sum\limits_{k\in I_1}\frac{1}{\lambda_k}+\sum\limits_{k\in J_1}\frac{1}{\nu_k}\right]}\right\}. \tag{3.10}$$

From condition (3.2) we obtain the following inequalities

$$\max\left\{\frac{(p_2-1)(n+m_2)}{4},\frac{(q_1-1)(n+m_2)}{4}\right\}\le$$
$$\le\min\left\{p_2'-\frac{n+m_2}{4},q_1'-\frac{n+m_1}{4}\right\}.$$

We choose

$$\lambda_k=2\chi\mu,\ k\in I_i,\ \nu_k=\chi\mu,\quad k\in J_i,\ i=1,2, \tag{3.11}$$

where $\chi>0$ and

$$\mu\in\Big[\max\left\{\frac{(p_2-1)(n+m_2)}{4},\frac{(q_1-1)(n+m_2)}{4}\right\},$$
$$\min\left\{p_2'-\frac{n+m_2}{4},q_1'-\frac{n+m_1}{4}\right\}\Big]. \tag{3.12}$$

It follows that

$$\frac{2}{\chi}+\sum_{k\in I_i}\frac{1}{\lambda_k}+\sum_{k\in J_i}\frac{1}{\nu_k}=\frac{1}{\chi}\left(2+\frac{n+m_i}{2\mu}\right);$$
$$\omega_{11}=-\frac{2p_2'}{\chi}+\frac{1}{\chi}\left(2+\frac{n+m_2}{2\mu}\right)=\frac{1}{\chi}\left(-\frac{2}{p_2-1}+\frac{n+m_2}{2\mu}\right);$$
$$\omega_{12}=-\frac{2p_2'}{\chi}+\frac{1}{\chi}\left(-\frac{2}{p_2-1}+\frac{n+m_2}{2\mu}\right);$$
$$\omega_{13}=\omega_{14}=\frac{1}{\chi}\left(-\frac{2p_2}{\mu\chi(p_2-1)}+2+\frac{n+m_2}{2\mu}\right);$$
$$\omega_{21}=-\frac{2q_1'}{\chi}+\frac{1}{\chi}\left(2+\frac{n+m_2}{2\mu}\right)=\frac{1}{\chi}\left(-\frac{2}{q_1-1}+\frac{n+m_2}{2\mu}\right);$$
$$\omega_{22}=-\frac{2q_1'}{\chi}+\frac{1}{\chi}\left(-\frac{2}{q_1-1}+\frac{n+m_2}{2\mu}\right);$$
$$\omega_{23}=\omega_{24}=\frac{1}{\chi}\left(-\frac{2q_1}{\mu\chi(q_1-1)}+2+\frac{n+m_2}{2\mu}\right).$$

From (3.12) it follows that

$$\omega_{ij}\le 0,\ i=1,2,\ \ j=1,2,3,4. \tag{3.13}$$

By letting $d\to\infty$ in (3.10), by virtue of (3.3), (3.6) and (3.13) it yields that

$$\int\limits_0^\infty\int\limits_{R^n}|u_1|^{p_2}\,dxdt+\int\limits_0^\infty\int\limits_{R^n}|u_2|^{q_1}\,dxdt\le C. \tag{3.14}$$

Further, by applying the Holder inequality, from (3.4), (3.5) we obtain

$$\sum_{j=1}^{2}\int_{R^n}\left(\varphi_j\left(x\right)+\psi_j\left(x\right)\right)\xi_j\left(x,0\right)dx+\int_0^\infty\int_{R^n}\left|u_1\right|^{p_2}\xi_2 dxdt+\int_0^\infty\int_{R^n}\left|u_2\right|^{q_1}\xi_1 dxdt\le$$

$$\le\left[\iint\limits_{d^2\le t^{\varkappa}+\sum\limits_{k\in I_2}|x_k|^{\lambda_k}+\sum\limits_{k\in J_2}|x_k|^{\nu_k}\le 2d^2}\left|u_2\right|^{q_1}dxdt\right]^{1/q_1}\cdot\left[\sum_{s=1}^{4}\gamma_{1s}\right]+$$

$$+\left[\iint\limits_{d^2\le t^{\varkappa}+\sum\limits_{k\in I_1}|x_k|^{\lambda_k}+\sum\limits_{k\in J_1}|x_k|^{\nu_k}\le 2d^2}\left|u_1\right|^{p_2}dxdt\right]^{1/p_2}\cdot\left[\sum_{s=1}^{4}\gamma_{2s}\right],\tag{3.15}$$

where

$$\gamma_{11}=\left[\iint\limits_{d^2\le t^{\varkappa}+\sum\limits_{k\in I_1}|x_k|^{\lambda_k}+\sum\limits_{k\in J_1}|x_k|^{\nu_k}\le 2d^2}\left|\frac{\partial^2\xi_1}{\partial t^2}\right|^{q_1'}dxdt\right]^{1/q_1'}=$$

$$=\left[\iint\limits_{1\le \tau^{\varkappa}+\sum\limits_{k\in I_1}|y_k|^{\lambda_k}+\sum\limits_{k\in J_1}|y_k|^{\nu_k}\le 2}\left|\frac{\partial^2\xi_1}{\partial \tau^2}\circ h_1\right|^{q_1'}dyd\tau\right]^{1/q_1'};$$

$$\gamma_{12}=\left[\iint\limits_{d^2\le t^{\varkappa}+\sum\limits_{k\in I_1}|x_k|^{\lambda_k}+\sum\limits_{k\in J_1}|x_k|^{\nu_k}\le 2d^2}\left|\frac{\partial\xi_1}{\partial t}\right|^{q_1'}dxdt\right]^{1/q_1'}=$$

$$=\left[\iint\limits_{1\le \tau^{\varkappa}+\sum\limits_{k\in I_1}|y_k|^{\lambda_k}+\sum\limits_{k\in J_1}|y_k|^{\nu_k}\le 2}\left|\frac{\partial\xi_1}{\partial \tau}\circ h_1\right|^{q_1'}dyd\tau\right]^{1/q_1'};$$

$$\gamma_{13}=\left[\iint\limits_{d^2\le t^{\varkappa}+\sum\limits_{k\in I_1}|x_k|^{\lambda_k}+\sum\limits_{k\in J_1}|x_k|^{\nu_k}\le 2d^2}\left|\Delta_{I_1}^2\xi_1\right|^{q_1'}dxdt\right]^{1/q_1'}=$$

$$=\left[\iint\limits_{1\le \tau^{\varkappa}+\sum\limits_{k\in I_1}|y_k|^{\lambda_k}+\sum\limits_{k\in J_1}|y_k|^{\nu_k}\le 2}\left|\Delta_{I_1}^2(\xi_1\circ h_1)\right|^{q_1'}dyd\tau\right]^{1/q_1'};$$

$$\gamma_{14} = \left[\iint\limits_{d^2 \le t^{\varkappa} + \sum\limits_{k\in I_1} |x_k|^{\lambda_k} + \sum\limits_{k\in J_1} |x_k|^{\nu_k} \le 2d^2} |\Delta_{J_1}\xi_1|^{q_1'}\, dxdt \right]^{1/q_1'} =$$

$$= \left[\iint\limits_{1 \le \tau^{\varkappa} + \sum\limits_{k\in I_1} |y_k|^{\lambda_k} + \sum\limits_{k\in J_1} |y_k|^{\nu_k} \le 2d^2} |\Delta_{J_1}(\xi_1 \circ h_1)|^{q_1'}\, dyd\tau \right]^{1/q_1'};$$

$$\gamma_{21} = \left[\iint\limits_{d^2 \le t^{\varkappa} + \sum\limits_{k\in I_2} |x_k|^{\lambda_k} + \sum\limits_{k\in J_2} |x_k|^{\nu_k} \le 2d^2} \left|\frac{\partial^2 \xi_2}{\partial t^2}\right|^{p_2'} dxdt \right]^{1/p_2'} =$$

$$= \left[\iint\limits_{1 \le \tau^{\varkappa} + \sum\limits_{k\in I_2} |y_k|^{\lambda_k} + \sum\limits_{k\in J_2} |y_k|^{\nu_k} \le 2} \left|\frac{\partial^2 \xi_2}{\partial t^2} \circ h_2\right|^{p_2'} dyd\tau \right]^{1/p_2'};$$

$$\gamma_{22} = \left[\iint\limits_{d^2 \le t^{\varkappa} + \sum\limits_{k\in I_2} |x_k|^{\lambda_k} + \sum\limits_{k\in J_2} |x_k|^{\nu_k} \le 2d^2} \left|\frac{\partial \xi_2}{\partial t}\right|^{p_2'} dxdt \right]^{1/p_2'} =$$

$$= \left[\iint\limits_{1 \le \tau^{\varkappa} + \sum\limits_{k\in I_2} |y_k|^{\lambda_k} + \sum\limits_{k\in J_2} |y_k|^{\nu_k} \le 2d^2} \left|\frac{\partial \xi_2}{\partial \tau} \circ h_2\right|^{p_2'} dyd\tau \right]^{1/p_2'};$$

$$\gamma_{23} = \left[\iint\limits_{d^2 \le t^{\varkappa} + \sum\limits_{k\in I_2} |x_k|^{\lambda_k} + \sum\limits_{k\in J_2} |x_k|^{\nu_k} \le 2d^2} \left|\Delta_{I_1}^2 \xi_2\right|^{p_2'} dxdt \right]^{1/p_2'} =$$

$$= \left[\iint\limits_{1 \le \tau^{\varkappa} + \sum\limits_{k\in I_2} |y_k|^{\lambda_k} + \sum\limits_{k\in J_2} |y_k|^{\nu_k} \le 2} \left|\Delta_{I_1}^2 (\xi_2 \circ h_2)\right|^{p_2'} dyd\tau \right]^{1/p_2'};$$

$$\gamma_{24}=\left[\iint\limits_{d^2\le t^{\varkappa}+\sum\limits_{k\in I_2}|x_k|^{\lambda_k}+\sum\limits_{k\in J_2}|x_k|^{\nu_k}\le 2d^2}\left|\Delta_{J_2}\xi_2\right|^{p_2'}dxdt\right]^{1/p_2'}=$$

$$=\left[\iint\limits_{1\le\tau^{\varkappa}+\sum\limits_{k\in I_2}|y_k|^{\lambda_k}+\sum\limits_{k\in J_2}|y_k|^{\nu_k}\le 2}\left|\Delta_{J_2}(\xi_2\circ h_2)\right|^{p_2'}dyd\tau\right]^{1/p_2'}.$$

By the construction of γ_{ij}, it is obvious that

$$0\le\gamma_{ij}\le C,\ \ i=1,2,\ \ j=1,2,3,4. \tag{3.16}$$

From (3.15) and the condition (3.6) we have

$$\lim_{d\to+\infty}\iint\limits_{d^2\le t^{\varkappa}+\sum\limits_{k\in I_2}|x_k|^{\lambda_k}+\sum\limits_{k\in J_2}|x_k|^{\nu_k}\le 2d^2}|u_1|^{q_1}\,dxdt=0. \tag{3.17}$$

and

$$\lim_{d\to+\infty}\iint\limits_{d^2\le t^{\varkappa}+\sum\limits_{k\in I_1}|x_k|^{\lambda_k}+\sum\limits_{k\in J_1}|x_k|^{\nu_k}\le 2d^2}|u_2|^{p_2}\,dxdt=0\ . \tag{3.18}$$

By letting $d\to\infty$ in (3.15), by virtue of (3.3) and (3.16)-(3.18), it yielded that

$$\sum_{j=1}^{2}\int\limits_{R^n}\left(\varphi_j\left(x\right)+\psi_j\left(x\right)\right)dx+\int\limits_0^{\infty}\int\limits_{R^n}|u_1|^{p_2}\,dxdt+\int\limits_0^{\infty}\int\limits_{R^n}|u_2|^{q_1}\,dxdt\ \le 0$$

Finally, taking into consideration the condition (3.6), we have that

$$u_1(t,x)=u_2(t,x)=0.$$

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