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# ENHANCEMENT OF CONTROLLABILITY OF VARIATOR-BASED FRICTION SYSTEMS WITH THE HELP OF PROACTIVE CONNECTION WITH DRIVE

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**Abstract**. Based on the method of mathematical modelling, the dynamic characteristic of mechanical systems is studied via a variator. It is indicated that, the application of proactive feedback can significantly improve the controllability of gears, including friction gears from the standpoint of tribological requirements.

## 1. Introduction

Recently, the tendency of widespread use of continuously variable transmission – variator has become noticeable in the sphere of structural design of the management of mechanical systems (figure 1). Variators are the drivers of motion control systems in various fields of mechanical engineering, robotics, and other automation equipment [1, p. 61; 2, p. 101; 3, p. 42]. Systems using variators as a transmission member of motion control in various mechanisms, often with better characteristics of motion control, while being devoid of high levels of resistance (main tribological component). What is more, it is known that, nominal modes of the operation of variator-based mechanisms allow a short-term slip of friction wheels, while the controlling functions of modern mechanisms are quite multifaceted.

The intensity of wear of frictional contact surfaces depends on a great number of factors, but the main factor among them is certainly the dynamic conditions of the operation of control mechanism. They are determined with lots of parameters of kinetics, such as shaft inertia moments, speed (performance) characteristics of servomotor and load. Solution of the compromise between the quantities of downforce – basic structural parameters of the system with variators – and increase of wear resistance of mechanisms are associated with the impulse control. A special attention is paid to the system in which a variator is an actuator of automatic control of the speed of gear rotation.

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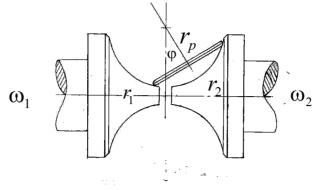


FIGURE 1. Schematic diagram of toroidal variator.

# 2. Objective of work and development of model:

In the current model study of the mechanical system, the main objective is to obtain transient characteristics reflecting the decrease of intensity of frictional slip depending on the clamping force on the friction wheel at different depths applying feedback.

Previously, we conducted a model study of mechanical characteristics of the mechanism with a variator model which represents the following system of differential equations [4, p. 363-370]:

$$\begin{cases} I_{1}\frac{d\omega_{1}}{dt} = M_{g} - sign\left(\eta_{1}\right) a_{2}P_{N}\varepsilon_{1}^{u}r_{1} - a_{2}P_{N}\varepsilon_{1}^{u+1}\omega_{1}^{-1} \\ I_{p}\frac{d\omega_{p}}{dt} = sign\left(\eta_{2}\right) a_{2}P_{N}\varepsilon_{2}^{u}r_{p} - sign\left(\eta_{1}\right) a_{2}P_{N}\varepsilon_{1}^{u}r_{p} \\ I_{2}\frac{d\omega_{2}}{dt} = sign\left(\eta_{2}\right) a_{2}P_{N}\varepsilon_{2}^{u}r_{2} - a_{2}P_{N}\varepsilon_{2}^{u+1}\omega_{2}^{-1} - M_{n}\left(t\right) \end{cases}$$
(2.1)

here  $\eta_1 = \omega_1 r_1 - \omega_p r_{@}$ ;  $\eta_2 = \omega_p r_{@} - \omega_2 r_2$ ,  $\varepsilon_i = |\eta_i|$ ;  $\omega_{@}, \omega_1, \omega_2, r_{@}, r_1, r_2 - respectively$  angular speed and radius of central, driving and driven disc;  $P_N - c$  lamping force on the shaft, a, u-parameters of design and materials of variator.

In the system (2.1) a slight value of inertia moment of central disc (roll)  $I_p$  is given compared with the inertial features of the mechanisms connected with the shaft variator  $I_1, I_2$ , i.e. assuming  $I_p \approx 0$ , it can be described as follows:

$$sign(\eta_2) a_2 P_N \varepsilon_2^u r_p - sign(\eta_1) a_2 P_N \varepsilon_1^u r_p = 0.$$

From which the following equation is formed:

$$\omega_p = \frac{\omega_1 r_1 + \omega_2 r_2}{2r_p}.\tag{2.2}$$

The equation (2.2) should be considered as a necessary kinematic equation in solving the system (2.1) which after reduction will consist of only two equations. On the other hand, taking into account the expression for the coefficient of the variator transmission  $k_{\varphi} = \frac{\omega_2}{\omega_1}$  that on the basis of the calculation of its structure (figure 1) is defined as [7, .167-370]:

$$k_{\varphi} = 0.87 \frac{1 + \sin \varphi}{1 - \sin \varphi}.$$
(2.3)

Then, using (2.3) for the mode in lack of slippage, it is possible to go to a model with a single equation:

$$\left(I_1 - 0.87 \frac{1 + \sin\varphi}{1 - \sin\varphi} I_2\right) \frac{d\omega_1}{dt} = M_g - F_c(r_1 + r_2) + M(t)$$
(2.4)

The mechanical system is in a state where the dynamic forces do not exceed the strength of friction clutch, with the motion described by the differential equation (2.4), while the forces of clutch are calculated using the following equation:

 $F_C = sign\left(\eta\right) a_2 P_N \varepsilon^u.$ 

In the moment of violation of condition  $F_C < sign(\eta) a_2 P_N \varepsilon^u$  clutch slippage occurs. In this state of the system, softness is acquired in the transient characteristic (smoothness in operation), nevertheless intensity of wear increases. This feature is an important advantage of the application of variator, and consequently the need for searching alternative solutions of mitigation of transient mode arises [5, p. 29].

One of such solutions of the problem can be the use of feedback through "transmission coefficient – torque".

The current regenerative scheme is realized through the establishment of any kind of connection between the torque and angle of rotation of the variator lever. In a number of drives, such a control of power flow can realize the impact upon the supply voltage. In another case, if the drive is equipped with DC motor, then the control of the angular moment is carried out by impacting on the magnetic flux in the excitation circuit. The figure 2 shows the structure of a closed loop system of control of power flow in connection with the function of control of the speed of driven disc. It is called a regenerative circuit which is built with the introduction of feedback.

Perturbation acting in the present situation as a power deficit can be compensated by increasing the amperage of winding. We use a small range of proportionality depending on the torque at a depth of feedback for the lever of variator control  $k_{\varphi}$ , i.e.

$$M_g(\omega_1, \varphi) = M_g(\omega_1)(1 + k_{\varphi}), \quad -\varphi_{\max} < \varphi < \varphi_{\max}$$
(2.5)

Taking the dependence of torque of drive at the speed of rotation of linearly decreasing function at a constant value of excitation current (control) [6, p. 432], i.e.

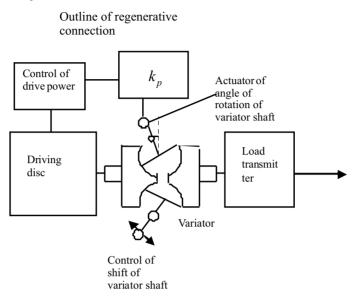
$$M_q(\omega_1) = M_{q0} - k_q \omega_1 \tag{2.6}$$

Using the formula, it is possible (2.6) to express as (2.5):

$$M_{g}(\omega_{1},\varphi) = (M_{g0} - k_{g}\omega_{1})(1 + k_{\varphi})$$
(2.7)

This structural change paves the way for significant improvement of the controllability of variator drives. In the quantity analysis of such improvements, the current work is focused on the model study for different values of parameters.

We consider two controllable actuator in one of which torque generated by drives is not connected with the value of dynamic resistance, that is, the feedback of ratio of variator transmitter is not included. It is clear that, in this case, drive torque is a function of only the rotational velocity, i.e. it is the place FIGURE 2. Regenerative communication of transmission coefficient – torque" which is used for the improvement of drive controllability.



for only the function of  $M_g \equiv M_g(\omega_1)$ . Another drive is covered by feedback  $M_g = (1 + k_{\varphi}\varphi)M_g(\omega_1)$ .

We suppose both the drives with a state of equilibrium  $\omega_2$ :  $\frac{d\omega_2}{dt} = 0, \varphi = 0, k = 1, t = t_0$  under the influence of control programs  $k(t) = k_{\varphi}\varphi(t); 0 < t < t_u$  find their own specific realization within the period of time  $t_i$  in the form of function  $\omega_2^*(\varphi(t)), \omega_2(\varphi(t))$  which are the realization programs of control over the angular speed in the appropriate drives covered by feedback and without it.

We introduce the functionals  $\delta^*[\varphi(t)]$ ,  $\delta[\varphi(t)]$  as an indicator of control that is based on the following two structures:

$$\delta^*[\varphi(t)] = \int_0^{t_u} \left(\frac{\omega_2 * (\varphi(t)) - \omega_0(1 + k_\varphi \varphi(t))}{\omega_0(1 + k_\varphi \varphi(t))}\right)^2 dt;$$
(2.8)

$$\delta[\varphi(t)] = \int_0^{t_u} \left(\frac{\omega_2(\varphi(t)) - \omega_0(1 + k_\varphi\varphi(t))}{\omega_0(1 + k_\varphi\varphi(t))}\right)^2 dt$$
(2.9)

here  $\omega_0(1 + \varphi(t))$  - the specified program of control in the interval of  $0 < t < t_u$ .

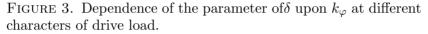
### 3. Arrangement of tasks:

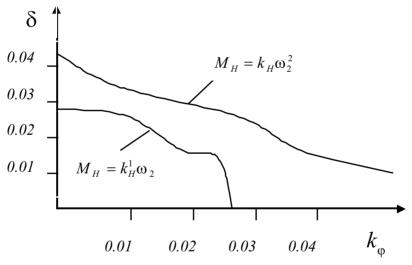
It is possible to put forward the following tasks of optimization: on the basis of structural parameters of variator ( $P_N, u$ ), kinetic parameter of mechanical system ( $\varphi$ ), as well as electromechanical ( $\varphi, M_{go}, k_g$ ) and load characteristic of drive  $M_H(\omega_2)$ , it is required to determine the optimal value of  $k_{\varphi}$  which enables the minimization of control program at the predetermined magnitude  $\delta [\varphi(t)]$ . This task can be implemented based on the computer simulation within the model of mechanical system (2.1).

#### 4. Results:

Based on the said model, a number of features were obtained that allow the assessment of several indicators of control of electric conductor with a variator for two strongly differentiating load characteristics,  $M_H = k_H^1 \omega_2$   $M_H = k_H \omega^2$  (load of dry and viscous friction respectively). As a program, sinusoidal law i.e.  $\varphi(t) = \varphi_m \sin \nu t$  was used.

It is visible from the figure 3 that, by increasing the coefficient of feedback  $k_{\varphi}$  there is a place for the improvement of the quality indicator of control of mechanical systems with a variator with respect to the criterion of  $\delta$ . Despite of the nonlinearity of load characteristics of the drive in the case of load of type of viscous friction  $(M_H = k_H \omega^2)$ , namely in this case a wider range of variation has the coefficient  $k_{\varphi}$ .





In the mode of operation of drive load on the linear law of variation at the speed of predetermined value ( $k_{\varphi} = 0.04 - 0.05$ ) destabilization of computing process was observed which simulated the mechanical system using the regenerative scheme of control of drive power.

The application of regenerative scheme of the control of drive power significantly impacted upon the frequency characteristics of mechanical system with a variator. Above all, it influenced the system response at highly harmonic control effects rendered by the lever of control of gear ratio of variator. Maximum frequency of control signal wherein the loss of control is not observed in the system, and in the mode of absence of the said regenerative connection it was  $\nu = 0.85 c^{-1}$ .

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The figure 4 shows the system response to the perturbation in the absence of regenerative connection at a frequency of  $\nu = 1.1 c^{-1}$  which is characterized in that the sinusoidal driving impacts are heavily distorted by transmitting the speed of rotation of drive shaft. The same frequency characteristic is indicated in the figure 5 which differs from the previous one in that it reflects the presence of the same regenerative communication (value of  $k_{\varphi} = 0.022$ ).

It is seen from the figure that, the system response to the harmonic control impact undergoes only a minor distortion which can practically be disregarded.

Study of the frequency characteristics brings the attention to the fact that in most cases, high oscillation disruption of frictional contact is observed in the system. The cause of periodic mode change of pure rolling of discs of frictional transmission with the mode of rolling with sliding is the hidden resonance arising from elastic conservatism which takes place in the mathematical model of the mechanical system. It manifests itself as an effect of accumulation of kinetic energy rotating the mechanical parts of the system upon the break of contact.

In the presence of regenerative connection of the control of gear drive, the clearance of such effects noticeably decreased.

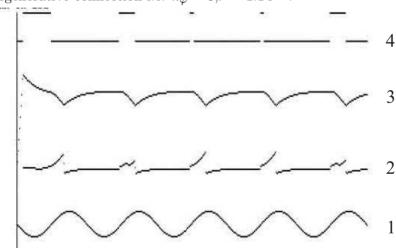
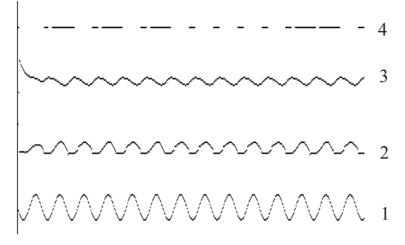


FIGURE 4. Characteristic of the control of the system without a regenerative connection i.e.  $k_{\varphi} = 0, \nu = 1.1 c^{-1}$ .

Thus, judging from the data of computer simulation we can surely come to conclusion that, the application of regenerative communications in the motion control with a variator can improve the mechanical indicators of the system to a great extent.

## 5. Key findings:

The use of the method of computer simulation can study different structures for the synthesis of motion control. In particular, the system has better mechanical FIGURE 5. Characteristics of controllability of the system in the case of use of regenerative communication with respect to the angle of rotation of variator lever where  $k_{\varphi} = 0.022, \nu = 1.1 c^{-1}$ .



characteristics in case the circuit closed-loop of control includes a regenerative connection of the moment resisting the flow of power heading from the network to the motor.

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