Proceedings of the Institute of Mathematics and Mechanics,
National Academy of Sciences of Azerbaijan
Volume 40, Number 2, 2014, Pages 132–140

CRACKING IN AN ANNULAR DISK WITH MIXED BOUNDARY CONDITIONS ON THE BOUNDARY

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Abstract. The mathematical description of cracking in an annular disk with mixed boundary conditions on the boundary is given. It is assumed that in the course of loading in the disk there exists a zone of weakened interparticle bonds of the material. The criterion of crack nucleation in an annular disk is formulated.

1. Problem statement

Let the cross section of the annular disk occupy in the plane $z = x + iy$ the annular domain S bounded from the outside by a circle of radius R_1 and from the within by a circle of radius R. Let's consider the stress-strain state in the annular domain S for the case when the external surface of the annular disk is fixed, i.e. is fastened with a rigid holder. For $r = R$ on the internal surface of the annular disk the external load is given.

In the course of operation, in the annular disk there will arise the prefracture zones (interlayers of the overstrained material), that we simulate as the areas of weakened interparticle bonds of the material. The interaction of the faces of these areas are simulated by the bonds between the prefracture zone faces having the given deformation diagram of the material bonds. Physical nature of these bonds and the size of prefracture zone wherein the interaction of the faces of interparticle bonds areas is realized, depend on the kind of the material [3].

Thus, the embryonic cracks are simulated by the prefracture zones with the bonds between the faces, that are considered as the areas of weakened interparticle bonds. As the prefracture zones of the annular disk material are small in comparison with the remaining part of the disk, they may be mentally replaced by the slits whose surfaces interact between themselves by some law corresponding to the action of the removed material of the disk. In the considered case the cracking is a process of a passage of the prefracture area to the area of broken bonds between the surface of the annular disk material.

Refer the annular disk to the polar system of coordinates $r\theta$ having chosen the origin of coordinates in the center of concentrical circles L_1, L of the radii R_1 and R. Let on the boundary of the annular area S there hold the following boundary

Key words and phrases. annular disk, zone of weakened interparticle bonds of the material, adhesion forces, cracking.

conditions (mixed boundary conditions):

$$\sigma_r - i\tau_{r\theta} = f_1(\theta) - if_2(\theta),\ |z| = R \quad (1.1)$$

$$\nu_r - i\nu_\theta = 0,\ |z| = R_1,$$

where $\sigma_r, \sigma_\theta, \tau_{r\theta}$ are the stress tensor components ν_r; and ν_θ are radial and tangential displacement of the points of the annular disk; $f_1(\theta)$ and $f_2(\theta)$ are external stresses acting on the internal circle $r = R$.

In the operator process, under the action of the force load, in the annular disk there will arise prefracture zones that will be simulated as the areas of weakened interparticle bonds of the material. It is assumed that the prefracture zones are oriented on maximal tensile stresses arising in the annular disk.

In the cross section of the disk consider the prefracture zone of length $2l_1$ located on the interval $|x_1| \leq l_1,\ y_1 = 0$. In the center of the prefracture zone locate the origin of the local system of coordinates $x_1O_1y_1$ x_1 whose axis α_1, coincides with prefacture zone line and forms the angle α_1 with the axis Ox $(\theta = 0)$.

The prefracture zone size is unknown beforehand and should be defined in the process of the problem solution. Under the action of external force loads on the annular disk in the bonds connecting the prefracture zone faces there will arise normal q_{y_1} and tangential $q_{x_1y_1}$ forces. For finding the stress-strain state of the disk with one rectilinear prefracture zone at loading by external load the joint solution of the equations of elasticity theory [4] under the disk loading conditions (1) is necessary. By loading the annular disk to these conditions we add conditions for the forces in the bonds

$$\sigma_{y_1} = q_{y_1}(x_1),\ \tau_{x_1y_1} = q_{x_1y_1}(x_1),\ y_1 = 0,\ |x_1| \leq l_1 \quad (1.2)$$

The main relations of the stated problem should be completed by the equation connecting the opening of the prefracture faces and the stresses in the bonds. This equation may be represented in the form

$$(\nu_1^+ - \nu_1^-) - i(u_1^+ - u_1^-) = \Pi(x_1, \sigma_1)\left[q_{y_1}(x_1) - iq_{x_1y_1}(x_1)\right], \quad (1.3)$$

where $(\nu_1^+ - \nu_1^-)$ and $(u_1^+ - u_1^-)$ are normal and tangential components of the opening of prefecture zone faces, respectively; $\Pi(x_1, \sigma_1)$ is the compliance of bonds dependent on the tension of bonds, for $\Pi = const$ the law of deformation of bonds is linear; $\sigma_1 = \sqrt{q_{y_1}^2 + iq_{x_1y_1}^2}$ is the modulus of vector of adhesion forces in the bonds. In the general case, the law of deformation of bonds is nonlinear and given.

For determining the values of the external load at which the crack nucleation happens (cracking) the problem statement should be complemented with the crack occurrence condition (criterion). A such a condition we accept the criterion of critical opening of the faces of weakened interparticle bonds zone of the material bonds (prefracture zones)

$$\left|(\nu_1^+ - \nu_1^-) - i(u_1^+ - u_1^-)\right| = \delta_c, \quad (1.4)$$

where δ_c is the characteristics of resistance of the annular disk material to crack initiation and is determined experimentally.

This extra condition permits to find the parameters of the annular disk at which the cracking happens.

2. Method of decision a problem

The enumerated differential equations and boundary conditions compose a closed system for determining the stresses and strains in the annular disk by loading with force load. Using the Kolosov-Muskheleshevili formula [4], boundary conditions (1), (2) are written in the form

$$\text{for } r=R \ \Phi(z)+\overline{\Phi(z)}-e^{2i\theta}[\overline{z}\Phi'(z)+\Psi(z)]=f_1(\theta)-if_2(\theta) \qquad (2.1)$$

$$\text{for } r=R_1 \ \Phi(z)-k\overline{\Phi(z)}-e^{2i\theta}[\overline{z}\Phi'(z)+\Psi(z)]=0,$$

$$\text{for } y_1=0,\ |x_1|\le l_1 \ \Phi(x_1)+\overline{\Phi(x_1)}+x_1\overline{\Phi'(x_1)}+\overline{\Psi(x_1)}=q_{y_1}-iq_{x_1y_1}, \qquad (2.2)$$

where $k=(3-\nu)/(1+\nu)$; ν is the Poisson ratio of the material.

We look for the complex potentials describing the stress-strain state of the annular disk with one rectilinear prefecture zone in the form [5, 2]

$$\begin{aligned}\Phi(z)&=\Phi_0(z)+\Phi_1(z)+\Phi_2(z),\\ \Psi(z)&=\Psi_0(z)+\Psi_1(z)+\Psi_2(z),\end{aligned} \qquad (2.3)$$

where

$$\Phi_0(z)=\sum_{k=-\infty}^{\infty}a_kz^k,\ \Psi_0(z)=\sum_{k=-\infty}^{\infty}b_kz^k, \qquad (2.4)$$

$$\Phi_1(z)=\frac{1}{2\pi}\int_{-l_1}^{l_1}\frac{g_1(t)dt}{t-z_1},$$

$$\Psi_1(z)=\frac{1}{2\pi}e^{-2i\alpha_1}\int_{-l_1}^{l_1}\left[\frac{\overline{g_1(t)}}{t-z_1}-\frac{\overline{T}_1e^{i\alpha_1}}{(t-z_1)^2}g_1(t)\right]dt, \qquad (2.5)$$

$$\Phi_2(z)=\frac{1}{2\pi}\int_{-l_1}^{l_1}\left[\frac{1-T_1\overline{T_1}}{\overline{T_1}T_2^2}e^{-i\alpha_1}\overline{g_1(t)}-\frac{1}{zT_2}e^{i\alpha_1}g_1(t)\right]dt,$$

$$\Psi_2(z)=\frac{1}{2\pi z}\int_{-l_1}^{l_1}\left\{\left[\frac{1}{zT_1}-\frac{1}{z^2}-\frac{1}{z^2T_2}+\frac{\overline{T_1}^2}{T_2^2}\right]e^{i\alpha_1}g_1(t)+\right.$$

$$\left.+\left[\frac{1-T_1\overline{T_1}}{z\overline{T_1}T_2^2}-\frac{1}{1-zT_1}-\frac{2(1-T_1\overline{T_1})}{T_2^3}\right]e^{-i\alpha_1}\overline{g_1(t)}\right\}dt; \qquad (2.6)$$

$$T_1=te^{i\alpha_1}+z_1^0,\ T_2=1-z\overline{T_1},\ z_1=e^{-i\alpha_1}(z-z_1^0)$$

In relations (9) and (10) $g_1(t)$ is a sought - for function characterizing the opening of the prefracture zone faces

$$g_1(x_1) = \frac{2\mu}{1+k}\frac{\partial}{\partial x_1}\left[\nu_1^+(x_1,0) - \nu_1^-(x_1,0) - i(u_1^+(x_1,0) - u_1^-(x_1,0))\right] \quad (2.7)$$

Here μ is the shear modules of the cylinder's material.

The complex potentials $\Phi_0(z)$ and $\Psi_0 z$ and the unknown sought - for function $g_1(x_1)$ should be determined from the boundary conditions on the contours $r = R$ and $r = R_1$ and on the faces of rectilinear prefracture zone.

For the complex potentials $\Phi_0(z)$ and $\Psi_0(z)$ we can write boundary conditions of problem (5) on the circular boundaries in the following form (12)

$$\Phi_0(\tau) + \overline{\Phi_0(\tau)} - e^{2i\theta}[\overline{\tau}\Phi_0'(\tau) + \Psi_0(\tau)] = f_1(\theta) - if_2(\theta) \text{ for } \tau = Re^{i\theta}, \quad (2.8)$$

$$\Phi_0(\tau_1) - k\overline{\Phi_0(\tau_1)} - e^{2i\theta}[\overline{\tau_1}\Phi_0'(\tau_1) + \Psi_0(\tau_1)] = -(f_1^*(\theta) - if_2^*(\theta)) \text{ for } \tau_1 = R_1e^{i\theta},$$

$$f_1^*(\theta) - if_2^*(\theta) = \Phi_*(\tau_1) - k\overline{\Phi_*(\tau_1)} - e^{2i\theta}[\overline{\tau_1}\Phi_*'(\tau_1) + \Psi_*(\tau_1)],$$

$$\Phi_*(z) = \Phi_1(z) + \Phi_2(z); \;\; \Psi_*(z) = \Psi_1(z) + \Psi_2(z) \quad (2.9)$$

The solution of boundary value problem is attained by the method of power series. For that it is necessary to expand the right sides of boundary conditions (12) in Fourier series. These expansions have the following form:

$$f_1(\theta) - if_2(\theta) = \sum_{k=-\infty}^{\infty} A_k e^{ik\theta} \text{ for } r = R \quad (2.10)$$

$$f_1^*(\theta) - if_2^*(\theta) = \sum_{k=-\infty}^{\infty} D_k e^{ik\theta} \text{ for } r = R_1,$$

where

$$A_k = \frac{1}{2\pi}\int_0^{2\pi}(f_1(\theta) - if_2(\theta))e^{-ik\theta}d\theta \;\; (k = 0, \pm1, \pm2, ...) \quad (2.11)$$

$$D_k = \frac{1}{2\pi}\int_0^{2\pi}(f_1^*(\theta) - if_2^*(\theta))e^{-ik\theta}d\theta$$

After integration by means of residue theory, for the coefficients D_k we find

$$D_n = -(1-k)\frac{1}{\pi}\int_{-l_1}^{l_1}\left[\frac{e^{i\alpha_1}g_1(t)}{T_1^{n+1}} - e^{i\alpha_1}\overline{T}_1^{n+1}g_1(t)\right]dt + C_n, \quad (2.12)$$

$$D_0 = -(1-k)\frac{1}{\pi}\int_{-l_1}^{l_1}\left[\frac{e^{i\alpha_1}g_1(t)}{T_1} - \frac{3}{2}e^{i\alpha_1}\overline{T}_1 g_1(t)\right]dt + C_0,$$

$$C_n = C_{n-2}' - 2C_{n-1}'' + C_n^* \;\; (n = 2, 3, ...),$$

$$C_1=-2C_0''+C_1^*,\ C_0=C_0^*+C_*,\ C_*=\frac{1}{\pi}\int\limits_{-l_1}^{l_1}T_1e^{-i\alpha_1}g_1(t)dt,$$

$$C_n'=\frac{n+1}{2\pi}\int\limits_{-l_1}^{l_1}e^{-i\alpha_1}g_1(t)\overline{T}_1^{n+1}dt\ (n=0,1,2,...),$$

$$C_n''=\frac{n+1}{2\pi}\int\limits_{-l_1}^{l_1}e^{-i\alpha_1}\overline{T}_1^{n}g_1(t)dt\ (n=0,1,2,...),$$

$$C_n^*=\frac{n+1}{2\pi}\int\limits_{-l_1}^{l_1}e^{-i\alpha_1}T_1\overline{T}_1^{n}\overline{g_1(t)}dt\ (n=0,1,2,...)$$

Satisfying by the functions (12) complex potentials (8) on the circular boundaries of the annular domain S, after some transformations we get an infinite system of linear algebraic equations with respect to the sought-for coefficients a_k and b_k

$$a_0+\overline{a}_0-b_{-2}R^{-2}=A_0,\ a_0-k\overline{a}_0-b_{-2}R_1^{-2}=-D_0, \tag{2.13}$$
$$(1-k)a_kR^k+\overline{a}_{-k}R^{-k}-b_{k-2}R^{k-2}=A_k,$$
$$(1-k)a_kR_1^k+k\overline{a}_{-k}R_1^{-k}-b_{k-2}R_1^{k-2}=-D_k,$$

The solution of the obtained system (17) is not difficult and has the following form:

$$a_0=\frac{A_0R^{-2}-D_0R_1^{-2}}{(1-k)R_1^{-2}-2R^{-2}}, \tag{2.14}$$

$$a_k=\frac{(1+k)(R_1^2-R^2)M_k-(kR_1^{-2k+2}+R^{-2k+2})\overline{M}_{-k}}{(1-k^2)(R_1^2-R^2)^2-(R^{-2k+2}+kR^{-2k+2})(R^{2k+2}+kR_1^{-2k+2})}\ (k=\pm2,+\pm3...),$$

$$M_k=-D_kR_1^{-k+2}-A_kR^{-k+2},\ b_{-2}R^{-2}=a_0+\overline{a}_0-A_0,$$
$$b_{k-2}R_1^{k-2}=(1-k)a_kR_1^k-k_0\overline{a}_{-k}R_1^{-k}+D_k,$$
$$a_{-1}=\frac{\overline{A}_1R}{1+k};\ b_{-1}=-\frac{kA_1R}{1+k};\ a_1=\frac{2a_{-1}(R^2-R_1^2)-\overline{M}_{-1}}{R^4+kR_1^4}$$

Satisfying by the functions (7)-(10) the boundary conditions on the prefecture zone faces for $y_1=0,|x_1|\le l_1$ we get a complex singular integral equation with respect to the unknown function $g_1(x_1)$:

$$\int\limits_{-l_1}^{l_1}[R(t,x_1)g_1(t)+S(t,x_1)\overline{g_1(t)}]dt=\pi f(x_1)\ |x_1|\le l_1, \tag{2.15}$$

where

$$f(x_1)=q_{y_1}-iq_{x_1y_1}+f_0(x_1),$$
$$f_0(x_1)=-[\Phi_0(x_1)+\overline{\Phi_0(x_1)}+x_1\overline{\Phi_0'(x_1)}+\overline{\Psi_0(x_1)}],$$

$$R(t,x_1)=\frac{e^{i\alpha_1}}{2}\left(\frac{1}{T_1-X_1}+\frac{e^{-2i\alpha_1}}{\overline{T}_1-\overline{X}_1}\right)$$

$$+\frac{e^{i\alpha_1}}{2}\left[\frac{1}{X_1(X_1\overline{T}_1-1)}+\frac{1-T_1\overline{T}_1}{T_1(1-T_1\overline{X}_1)^2}+\right.$$

$$\left.+e^{-2i\alpha_1}\frac{(T_1\overline{T}_1-1)(2X_1T_1\overline{X}_1^2-3\overline{X}_1T_1+1)+T_1\overline{X}_1(1-\overline{X}_1T_1)^2}{T_1\overline{X}_1^2(T_1\overline{X}_1-1)^3}\right],$$

$$S(t,x_1)=\frac{e^{-i\alpha_1}}{2}\left[\frac{1}{\overline{T}_1-\overline{X}_1}-\frac{T_1-X_1}{(\overline{T}_1-\overline{X}_1)^2}e^{-2i\alpha_1}\right]+$$

$$\frac{e^{-i\alpha_1}}{2}\left\{\frac{1}{\overline{X}_1(T_1\overline{X}_1-1)}+\frac{1-T_1\overline{T}_1}{\overline{T}_1(1-X_1\overline{T}_1)^2}+\right.$$

$$\left.+e^{-2i\alpha_1}\left[\frac{1}{\overline{T}_1\overline{X}_1^2}+\frac{X_1\overline{X}_1(1-2\overline{X}_1T_1)+3\overline{X}_1T_1-2}{\overline{X}_1^3(1-\overline{X}_1T_1)^2}\right]\right\},$$

$$X_1=x_1e^{i\alpha_1}+z_1^0$$

To the singular integral equation for the internal prefracture zone it is necessary to add the extra equality

$$\int_{-l_1}^{l_1}g_1(t)dt=0 \tag{2.16}$$

Applying the change of variables and quadrature formula to singular integral equation (19) and to additional condition (20), we get the system of m algebraic equations for finding the m unknowns $g_1(t_m)=\nu_1^0(t_m)-iu_1^0(t_m)$

$$\frac{1}{M}\sum_{m=1}^{M}l_1\left[\varphi_0(t_m)R(l_1t_m,l_1\eta_r)+\overline{\varphi_0(t_m)}S(l_1t_m,l_1\eta_r)\right]=f(\eta_r) \tag{2.17}$$

$$\sum_{m=1}^{M}\varphi_0(t_m)=0,\ \ (r=1,2,...,M-1)$$

If in (21) we pass to complexly-conjugate variables, we get one more M algebraic equations. The stresses in the annular disk under force loading are everywhere restricted, therefore the solution of the singular integral equation should be sought in the class of everywhere bounded functions. Thus, it is necessary to add to the system of algebraic equations the stress boundedness conditions in the vicinity of the prefracture zone tips. By adding these conditions

$$\sum_{m=1}^{M}(-1)^m\varphi_0(t_m)ctg\frac{2m-1}{4M}\pi=0, \tag{2.18}$$

$$\sum_{m=1}^{M}(-1)^{M+m}\varphi_0(t_m)tg\frac{2m-1}{4M}\pi=0,$$

we get complex algebraic system (21), (22) for finding M unknowns $\varphi_0(t_m), (m = 1, 2, ..., M)$ and the prefracture zone sizes.

The right sides of the system (19) contain the unknown values of normal $q_{y_1}(x_1)$ and tangential $q_{x_1y_1}(x_1)$ stresses at the nodal points of zones of the weakened interparticle bonds of the material. For determining them we use additional equation (3):

$$g_1(x_1) = \frac{2\mu}{1+k}\frac{d}{dx_1}[\Pi(x_1, \sigma_1)(q_{y_1}(x_1) - iq_{x_1y_1}(x_1))] \tag{2.19}$$

This complex equation is used to determine the unknown stresses $q_{y_1}(x_1)$ and $q_{x_1y_1}(x_1)$ in the bonds between the prefracture zone faces. For constructing the missing equations that are used to determine the stresses in the bonds between the prefracture faces, it will be necessary to fulfill conditions (23) at the nodal points t_m contained in the prefracture zone. As a result we get two more systems from M equations for determining the approximate values $q_{y_1}(t_m)$ and $q_{x_1y_1}(t_m)$ $(m = 1, 2, ..., M)$. Therewith we use the method of finite differences.

Because of the unknown size of the prefracture zone length l_1 the algebraic system (21)-(23) became nonlinear even for linear bonds. The obtained algebraic systems (18), (21)-(23) are connected and should be solved jointly. For solving the obtained algebraic systems, the method of sequential approximations was used in the following form. The system was solved at some definite value of l_{1*} with respect to the unknowns $\nu_1^0(t_m),\ u_1^0(t_m),\ q_{y_1}(t_m),\ q_{x_1y_1}(t_m)$ $(m = 1, 2, ..., M)$. Then the value of l_{1*} and the found values $\nu_1^0(t_m),\ u_1^0(t_m),\ q_{y_1}(t_m),\ q_{x_1y_1}(t_m)$ were substituted into (22), i.e. into the unused equation of the combined system. The taken value of the parameter l_{1*} and the corresponding values $\nu_1^0(t_m),\ u_1^0(t_m),$ $q_{y_1}(t_m), q_{x_1y_1}(t_m)$, generally speaking, will not satisfy equation (22)of the system, therefore, choosing the values of the parameter l_{1*} the calculations were repeated over and over again until the equations (22) of the system were satisfied, with the given accuracy. The algebraic system at each approximation was solved by the Gauss method with selection of the principal element. For the case of the nonlinear law of deformation of bonds, for finding the stresses in the prefracture zone, the iterative algorithm similar to the method of elastic solutions [1] was used.

The obtained systems of equations with respect to $a_k,\ b_k,\ g_1^0(t_m),\ q_{y_1}(t_m),$ $q_{x_1y_1}(t_m), (m = 1, 2, ..., M)$ permit under the given external force loading to find the stress-strain state of the annular disk in the presence of a rectilinear prefracture zone.

Using the solution of the problem, we calculate the displacements on the prefracture zone faces for $x_1 = x_0$:

$$-\frac{1+k}{2\mu}\int\limits_{-l_1}^{x_0} g_1(x_1)dx_1 = \nu_1^0(x_0, 0) - iu_1^0(x_0, 0),$$

where $\nu_1^0(x_0, 0) = \nu_1^+(x_0, 0) - \nu_1^-(x_0, 0),\ u_1^0(x_0, 0) = u_1^+(x_0, 0) - u_1^-(x_0, 0)$

Now using the change of integration variable and replacing the integral by means of the Gauss quadratic formula, we find

$$-\frac{1+k}{2\mu}\frac{\pi l_1}{M}\sum_{m=1}^{M} g_1(t_m) = \nu_1^0(x_0,0) - iu_1^0(x_0,0),$$

where M_1 in the amount of nodal points belonging to the segment $(-l_1, x_0)$.

Obviously, the breaking of interparticle bonds of the disk material will occur in the middle part of the prefracture zone, i.e. $M_1 = 1/2M$.

Separating the real and imaginary parts in the previous relation, we find

$$\nu_1^0(x_0,0) = -\frac{1+k}{2\mu}\frac{\pi l_1}{M}\sum_{m=1}^{M_1}\nu_1^0(t_m),\ u_1^0(x_0,0) = -\frac{1+k}{2\mu}\frac{\pi l_1}{M}\sum_{m=1}^{M_1}u_1^0(t_m)$$

Thus, for the displacement vector modulus on the prefracture zone faces for $x_1 = x_0$ we get

$$V_{01} = \sqrt{[\nu_1^+(x_0,0) - \nu_1^-(x_0,0)]^2 + [u_1^+(x_0,0) - u_1^-(x_0,0)]^2} = \frac{1+k}{2\mu}\frac{\pi l_1}{M}\sqrt{A_1^2 + B_1^2},$$

$$A_1 = \sum_{m=1}^{M_1}\nu_1(t_m),\ B_1 = \sum_{m=1}^{M_1}u_1(t_m)$$

Consequently, the condition determining the critical value of the external force load at which the cracking happens in the annular disk, will be

$$\frac{1+k}{2\mu}\frac{\pi l_1}{M}\sqrt{A_1^2 + B_1^2} = \delta_c \tag{2.20}$$

The joint solution of the obtained equations (18), (21)-(24) permits to determine under the given characteristics of the annular disk the critical value of the external force load and the prefracture zone size l_1^c for the limit equilibrium state. The velocity of strain energy consumption $G_n(l_1^c)$ obtained from this solution is considered the energetic characteristics of fracture resistance, i.e.

$$G_c = G_n(l_1^c)$$

Using the ultimate values δ_c and G_c we can isolate different regimes of prefracture zone equilibrium and crack nucleation in the annular disk under mixed boundary conditions. For example, subject to the conditions

$$G_b(l_1) \geq G_c,\ V_{01}(x_0) < \delta_c$$

there happens advance of the prefracture zone tip without breaking of interparticle bonds of the material, i.e. in this case the crack doesn't initiate. This state of prefracture zone growth is interpreted as adaptability of the annular disk to the given level of the external load acting on the internal surface of the disk in the loading process. The growth of the prefracture zone tip with simultaneous breaking of interparticle bonds of the material on the prefracture zone faces will happen subject to the following conditions

$$G_b(l_1) \geq G_c,\ V_{01}(x_0) \geq \delta_c$$

while subject to the conditions

$$G_b(l_1) < G_c,\ V_{01}(x_0) \geq \delta_c$$

the breaking of interparticle bonds of the material happens without advance of the prefracture zone tip, i.e. in this case the crack initiates, and the prefracture zone size decreases. If the following conditions

$$G_b(l_1) < G_c,\ V_{01}(x_0) < \delta_c,$$

are fulfilled, the state of the prefracture zone tip will not change.

Thus, the analysis shows that value of the external load and critical parameters δ_c, G_c determine the cracking's character.

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Received: May 22, 2014; Accepted: October 10, 2014