

ON HISTORY OF DEVELOPMENT OF OPTIMAL CONTROL THEORY IN AZERBAIJAN

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Abstract. In this review paper we adduce the main scientific results in the field of mathematical theory of optimal control obtained by Azerbaijan scientists for the last 50 years. The material is grouped around the following themes: Pontryagin's maximum principle, singular control, existence of optimal controls; discrete systems; identification; computational optimization algorithms; parametrization method for constructing optimal regulator; intellectual control; determinate and stochastic optimal control problems; optimization of discrete and differential inclusions.

As is known optimal control theory is one of the significant and actual branches of mathematics and has various applications in engineering, economics, control, natural science and also to mathematics itself.

In mankind's history the choice of the best and optimal among the all possible situations always was a great of interest. To the middle of the XX century such problems were solved by means of classical variational calculus methods [1, 2].

At the early fiftieth of the XX century it was observed that many problems of economics, space navigation, rocket dynamics, etc. cannot be solved by these methods. Just at that time this theory was widened receiving the name of "optimal control theory". The principles of this theory were founded by the group of mathematicians led by academician L.S. Pontryagin. The basic result of this theory is "Pontryagin's maximum principle" being the first order necessary optimality condition [3].

In Azerbaijan, in the sixtieth of the XX century, for the first time academician Zahid Khalilov and his followers began to study some linear problems of optimal control in Hilbert space by functional analysis methods [4, 5].

It should be noted that the founder of the school of mathematical theory of optimal control in Azerbaijan was a corresponding member of the Academy of Sciences of Azerbaijan, professor Koshkar Akhmedov who has made a valuable contribution to this orientation. Just under his direct guidance at the early sixtieth of XX century at the chair "Differential and Integral Equations" of Baku State University it was established a seminar "Mathematical theory of optimal control". Kazim Hasanov, Mamed Yagubov, Murguzali Aliyev, Aladdin Shamilov, Seyidali Akhiyev, Misir Mardanov, Akper Mamedov, Telman Malikov, and others were the active participants of this seminar. The first results of the participants of this

seminar, devoted to establishment of the analogue of Pontryagin's maximum principle in the problems of optimal control for the ordinary dynamical systems with delay, Goursat-Darboux systems with delay and systems of equations appeared in scientific journals at the late sixtieth and early seventieth [6,7,11].

In 1972, on initiative professor Koshkar Akhmedov, in Azerbaijan the first All-Union Conference "Problems of Control" was held and the prominent specialists of optimal control theory as A.I. Egorov, V.F. Krotov, F.M. Kirillova, V.G. Boltyanskii, T.K. Sirazetdinov, F.P. Vasil'ev and others attended this conference.

For the last 50 years, in optimal control field the Azerbaijan mathematicians have defended 20 doctor and 100 Ph.D. degree dissertations. At present, these scientists successfully continue their research and teaching in Azerbaijan and also in other countries.

At this review paper we adduce the main scientific results obtained by Azerbaijan scientists for the last 50 years in the field of mathematical theory of optimal control that was developed by some schools.

As noted, the first results in Azerbaijan in this direction belong to representatives of the school led by professor Koshkar Akhmedov that has made a valuable contribution to this direction. Under guidance of professors Koshkar Akhmedov and Kazim Hasanov, in 1973 for the first time in this field, in Azerbaijan Seyidali Akhiyev and Misir Mardanov defended their candidate degree dissertations [9-10]. Continuing the traditions of this school professor Kazim Hasanov proved the existence of optimal control and obtained necessary and sufficient conditions for optimality and also higher order necessary conditions for singular controls in optimal control problems described by integro-differential equations with delayed arguments [11, 12, 13, 14, 15]. He obtained the sign of controllability for linear impulse systems with a delay argument [16]. Existence and uniqueness of the solution of the Goursat-Darboux problem for the second order linear equations with delay argument and with impulse were studied. For such equations, optimal control problems were studied. Existence of optimal control in problems with nonlinear parabolic equations with non-classical boundary conditions, and also for systems described by the first order partial equations proved [17, 18, 19]. First order necessary conditions, sufficient conditions were obtained, the minimal energy problem was solved by means of moment problem, and controllability problem was studied [20, 21, 22].

The next representative of this school professor Mamed Yagubov defended the Doctor's degree dissertation on optimal control in 1991. For the processes described both by elliptic and hyperbolic type equations, he derived necessary optimality conditions of the first and second orders; necessary optimality conditions at additional functional constraints of equality and inequality types; necessary conditions for optimality of generalized control; by means of the introduced notion of a generalized control dependent on several parameters, sliding modes were studied, extended problems were constructed, theorems on the existence of optimal control were proved [23-31].

The representative of this school professor Seyidali Akhiyev obtained necessary optimality conditions in the form of L.S. Pontryagin's maximum principle in some optimal processes, described by the different classes of functional-differential

equations. In Seyidali Akhiev's works for the optimal control problems described by ordinary differential equations and Goursat-Darboux systems with neutral type delays, V.F. Krotov type sufficient conditions of optimality were established. The conjugate equations were constructed in the integral form for representation of the solution of a local and nonlocal boundary value problem at weak conditions on the coefficients of the equation. For similar problems necessary optimality conditions in the form of L.S. Pontryagin's maximum principle were proved by means of this representation [32-37].

Misir Mardanov being a doctor of physical-mathematical sciences since 1989, in his Ph.D. dissertation studied the problems of control of systems described by neutral type delay argument integro-differential equations with functional constraints, and proved Pontryagin's maximum principle-type necessary optimality conditions and also the existence of optimal control [10]. Further new, stronger necessary optimality conditions were obtained for optimal control problems described by the systems of differential equations with delay argument [38]. In his doctor's degree dissertation Misir Mardanov developed unique method for proving Pontryagin's maximum principle, and also second order necessary and sufficient conditions of optimality for a wide class of optimal control problems with concentrated and distributed parameters at different constraints. Suggested by him method allowed one to overcome some principal difficulties arising in application of the previously known methods. Existence of delays in controls that a priori are not supposed to be commensurable, absence of a priori assumption on normality of the studied extremal, existence of phase constraints, nondifferentiable dependence of right sides of equations on time, the end of which as a rule is not supposed to be fixed, are among these difficulties [39,40,41]. For the last years, Misir Mardanov's scientific activity is in the field of discrete control systems. Taking into account specifics of these systems and new properties of optimal controls, a strong optimality condition of linearized-type was obtained, and optimality of quasi-singular controls were studied [42]. For the above problem more strong necessary optimality conditions were obtained by introducing the notion of zero, first, second variations of the quality functional [43]. A discrete optimization problem with rather general initial data (without assumptions as convexity and smoothness) is considered, and taking into account the specifics of the discrete system, a necessary optimality condition that is not formulated by the Hamilton-Pontryagin function, is obtained[44].

The next representative of this school Kamil Mansimov defended doctor's degree dissertation in 1994. Based on the idea of increments method he has suggested a new mathematical technique for obtaining necessary conditions for optimality of singular controls and derivation of second order necessary optimality conditions for a rather wide class of problems of control of discrete and continuous time [45-48]. Multipoint necessary optimality conditions of controls singular in the sense of Pontryagin's maximum principle, quasi-singular and singular controls in the classic sense in the systems with delay (both in continuous and discrete time), in canonical hyperbolic systems of first order in Goursat-Darboux systems, in discrete two-parameter systems [47-54] were obtained. Krotov-type

sufficient optimality conditions in discrete and two-parameter systems were obtained. Necessary optimality conditions in the processes described by Volterra-type integral equations were derived. Necessary optimality conditions were obtained in discretely-continuous systems and in control problems described by integro-differential equations (jointly with M.J. Mardanov), Volterra-type difference equations [54, 55]. Stepwise optimal control problems with concentrated and distributed parameters were studied [56, 57].

Another representative Hamlet Kuliyeu defended doctor's degree dissertation in 1996. He proved existence theorems of optimal control and derived necessary optimality conditions in the processes described by first and second order hyperbolic equations with control functions at higher coefficients and right parts of equations [58, 59, 62, 64]. Control problems for the second and fourth order linear hyperbolic equations were studied [60,63]. Some ill-posed boundary value problems for elliptic and hyperbolic equations of second order were researched by means of the methods of optimal control theory [61].

In the works of Telman Melikov being a doctor of physical-mathematical sciences since 2005, the problems of optimal control of systems of differential equations with a contagion, Goursat-Darboux systems and also discrete systems were studied. As is known, when studying singular controls for problems described by ordinary differential equations, R. Gabasov [65] has offered a method based on matrix impulses. However, in the paper [66] T. Melikov showed that this method is not applicable to systems with contagion. At the same time, in the papers [66,67], suggesting a new method, R.Gabasov type necessary optimality conditions were obtained. In [65], by introducing a new conjugate equation expressed by matrix variations of the Riemann function, the analogue of Pontryagin's maximum principle and stronger optimality condition of singular controls in Goursat-Darboux systems were obtained. First a new technique was suggested and an analogue of Pontryagin's maximum principle was obtained for controllable nonlinear systems with a neutral type contagion [68]. Following this technique, in the papers [67] the Kelly conditions, R. Gabasov-type conditions and equality-type conditions were obtained for controllable nonlinear systems with a neutral-type contagion. In the papers [68,70,71], carrying out quality investigations of singular (in the classical sense) and quasi-singular controls in optimal systems with delay, Kelly, Koppa-Moer, R. Gabasov and equality-type optimality conditions were obtained in the recurrent form. In the paper [72], a new sequence of necessary optimality conditions for singular controls was obtained. In [67,73], for Goursat-Darboux controllable systems, the notion of intermediate functionals are introduced and different necessary optimality conditions including Kelly, Koppa-Mayer, R. Gabasov and equality-type conditions are obtained in the recurrent form.

The next representative of this school Akper Mamedov defended PhD dissertation in 1979. He has studied a problem on taking the system described by linear partial equations from the initial state to the given final state in the shortest time. The stated problem was reduced to L-problem of moments, the existence and uniqueness of the solution of the stated problem was proved [74]. He also proved the existence and uniqueness of the solution for a similar optimal control problem in Hilbert's vector space. The approximate solution of an optimal

control problem was constructed by means of the moment problem [75]. In one-dimensional and many-dimensional cases, the existence and uniqueness of the solution of an optimal control problem was proved for systems described by a hyperbolic equation [76,77,79]. He has also studied optimal control problems for systems described by a hyperbolic equation with additional conditions [79].

Candidate of physical-mathematical sciences Yagub Sharifov in his paper [80] for the first time obtained first and second order optimality conditions for an optimal control problem with two-point boundary conditions. Later, this result was obtained for an optimal control problem with integral conditions. In subsequent years, similar results were obtained for different optimal control problems with various nonlocal boundary conditions [81-85]. In [86] Pontryagin's maximum principle was proved for an optimal control problem with multipoint boundary conditions.

The representative of this school Shakir Yusubov has proved theorems on the existence of optimal control and obtained necessary optimality conditions for Goursat-Darboux systems at local and nonlocal boundary conditions [87,88]. Necessary optimality conditions of singular controls for systems with impulse actions were proved [89]. The definition of singular, by the components, control was introduced and necessary optimality conditions for such controls were obtained [90,91].

Bakir Yusifov that has defended PhD dissertation in 1983, has obtained necessary optimality conditions of controls singular at the control point, in the systems with delay. Some second order optimality conditions were obtained and quasi-singular controls in the systems with delay were studied [93]. Existence of optimal controls in problems of control of the systems of integro-differential equations with delay were researched [92,94].

The school under the guidance of academician of the Azerbaijan National Academy of Sciences Fikret Aliyev is engaged in applied problems of optimal control theory. In his works, F. Aliyev has developed parametrization methods for constructing optimal regulator in discrete case, suggested new, more effective methods for constructing optimal trajectory and factorization of fractional-rational and matrix polynomials [95]. F. Aliyev and his followers first suggested the methods of modeling, control and stabilization of the gas lift process [96].

Doctor of physical-mathematical sciences, professor Misreddin Sadigov defended doctor's degree dissertation in 1993. He has studied subdifferentiability of an integral and terminal functional given on Sobolev space, obtained necessary and sufficient conditions of extremum for the Bolts generalized problem in Sobolev spaces, necessary and sufficient conditions of first and second orders extremum for differential inclusions with Bolts criteria. Necessary conditions for differential inclusions when optimal trajectory is not internal, were obtained [99,101]. For the first time, necessary condition for extremal problems for multi-dimensional differential inclusions was studied. Dependence of solutions of many-dimensional inclusions on perturbation, an extremal problem for many-dimensional differential inclusions [97,98], dependencies of the solutions of integral inclusions on perturbations and extremal problem for integral inclusions were researched [97].

Generalized solution of extremal problems was defined and the existence of subdifferential, approximate subdifferential and their properties was considered. Necessary second order conditions of extremum in the presence of constraints were obtained [98,101]. Different definitions of a higher order subdifferential were given, their properties were studied. Geometrical aspects of a higher order subdifferential were considered. n -subdifferential was defined, its properties were studied [97,99,101]. Higher order necessary and sufficient conditions in the presence of constraints were obtained [97,99,101]. n -convex, n -positive homogeneous function and n -convex set were determined and their properties were studied. Theorems on continuation of n -linear functional were proved [99,100,101].

Yusif Gasimov having defended his Doctor of Sciences degree dissertation in 2010, in his works considered mainly different shape optimization problems for eigenvalues of operators and suggested new effective methods for solving them [102]. In his works he first found the formulas for the variation of the eigenvalues with respect to domain, investigated very important properties of the eigenvalues relatively domain, obtained principally new formula for the eigenvalues of the Schrodinger operator, generalized these results for the p -Laplacian. By means of these methods he first introduced the definition of s -function and suggested a scheme for solving the inverse spectral problem on design of domain according to the given set of these functions [103]. He investigated different optimization problems for domain-dependent functionals and suggested numerical algorithms for solving them [104].

Another representative of this school Murad Imanov has defended his doctor's degree dissertation in 2014, and in his papers he developed a new approach, a method of similar solutions for studying optimal control problems with phase constraints [105]. By means of this method, he reduced studies of such problems to problems without phase constraints [106]. Therewith, a conjugate function is absolutely continuous or non-trivial on all time segments and this excludes the case of degeneration of the problem. The case when conjugate system of equations is homogeneous, though in the problem statement there is active phase constraint, is also considered [105].

The next representative of this school Mutallim Mutallimov defended doctor's degree dissertation in 2013. He constructed and justified a mathematical model of sucker-rod installation; numerical-analytical method for solving an optimal control problem with multipoint boundary conditions [107]; a numerical-analytical method for solving a problem of optimal control of gaslift operation of oil wells [108]; a numerical-analytical method for solving a problem of optimal stabilization of the work of sucker-rod installation; a new diagnostic method for defining malfunctions of sucker-rod installations was developed based on neural nets WTA [109].

Significant contribution to development of optimal control theory was made by the school led by prof. Asaf Iskenderov [110-131]. He gave new statements of inverse problems on identification of coefficients of linear and quasilinear equations of mathematical physics and also optimal control of coefficients of these equations and their discrete analogues, developed theory of well-posedness and numerical solution of such problems. He first studied well-posedness of a statement, developed a regularizing algorithm for solving the inverse problem on identification

of coefficients of differential equations dependent on their solution, considered loaded differential equations of mathematical physics, developed a qualitatively new theory and computing methods for solving boundary value problems for basic types of these equations. He solved the problems of optimal control of coefficients and boundary of domain for linear and quasilinear differential equations of mathematical physics, and also differential-operator equations with quality functional encountered in urgent scientific-parabolic problems. Necessary and sufficient conditions for optimality and well-posedness were proved. Algorithms of numerical solution that were applied to model and real scientific-technical problems, were developed. New quality functionals were considered for problems of optimal control of economic-ecological systems, and existence of the property of determined chaos was established.

A.D. Iskenderov's follower professor Gabil Yagubov who defended doctor's degree dissertation in 1994, in his papers [116,118,127,129,131-133], studied well-posedness of problems of optimal control described by Schrodinger's linear and quasilinear equations with controls in the coefficient of these equations and derived necessary optimality conditions. Numerical methods for solving the problems of optimal control of the coefficients of Schrodinger's linear and quasilinear equations were developed.

Doctor of phys. math. sci. Agaddin Niftiyev [115,131,135-137] in variation calculus problems with variable integration domain and optimal control for elliptic equations with an unknown boundary has obtained necessary optimality conditions and developed an algorithm for numerical solution of this class of optimal control problems. He proved theorems on the existence of solutions for some nonconvex problems of variational calculus.

In the papers [124,125,128,138-148], another representative of this school prof. Rafig Tagiyev that defended doctor's degree dissertation in 2011, studied optimal control problems for systems with distributed parameters with the controls in coefficients of the operator of state (especially in its principal part). He considered from unique positions the problems of control in the coefficients for different type partial equations both linear and nonlinear. Well-posedness of the statement of appropriate optimal control problems was studied and sufficient conditions for the existence of first variations and Frechet differentiability of aim functional were found, the expressions for their first variations and derivatives were found. Necessary optimality conditions in the form of the maximum principle and variational inequalities were justified. Convergence of approximation and regularization methods for the considered optimization problems was established.

Studies [149,150] of prof. A. Iskenderov's follower, Natik Ibrahimov who has defended doctor's degree dissertation in 2014 are devoted mainly to identification and optimal control problems for equations of quantum quasi-optics. He proved existence and uniqueness of the solution of identification and optimal control problems for linear and quasilinear equations of quasi-optics, found sufficient conditions for Frechet differentiability of aim functional, established necessary optimality conditions for solving the considered problems in the form of Pontryagin's maximum principle and variational inequality.

A representative of this school Shovkat Bakhishov [151] studied problems of identification and optimal control for linear equations of thermoelasticity, established necessary optimality conditions in the form of the maximum principle and variational inequalities.

Corresponding-member of NAS of Azerbaijan Aydzadeh Kamil defended doctor's degree dissertation in 1991. The basic orientation of his studies is development and justification of numerical methods for solving different classes of problems of finite-dimensional optimization and optimal control of objects with concentrated and distributed parameters, their application in solving practical problems in different assignment control systems. He developed a decomposition method to mathematical simulation and optimization of parameters of complex technical and technological objects. The suggested method was applied to the solution of many problem of optimization and optimal control of objects with concentrated and distributed parameters, for optimal synthesis of parameters of complex plane and space mechanisms [152-153]. The results were used for determining zonal values of parameters in inverse problems of hydrogasdynamics and pipeline transportation of raw material [154-156]. The problems of optimization of allocation and optimal control of the motion of concentrated sources in the systems with distributed parameters in different classes of control action [157-159] were solved. Inverse problems on definition of places of raw material loss, hydraulic resistance coefficient of the problems of optimal control of transient processes arising in oil-gas pipeline nets of complex structure and described by the systems of large number of hyperbolic type differential equations [160-162] were investigated. Necessary optimality conditions were obtained and numerical methods for solving inverse problems and optimal control problems described by loaded differential equations with ordinary and partial derivatives with nonlocal unseparated pointwise and integral conditions were suggested [163-167]. For problems of synthesis of optimal control with feedback for a bar (plate) heating process in a furnace, necessary optimality conditions were obtained, a numerical method for the solution was obtained [168,169]. Packets of programs for solving unconditional, conditional, global, multicriterial optimization problems that were used in solving many practical problems were developed [170,171,172,173].

Beginning from the middle seventieth of the XX century, at the Institute of Cybernetics of the Academy of Sciences of Azerbaijan a seminar under the guidance of acad. Jalal Allahverdiyev on determined and stochastic optimal control problems, whose participants were Agamirza Bashirov, Nazim Mahmudov and others, get started its work. The representative of this school Agamirza Bashirov defended doctor's degree dissertation in 1991. His main results belong to theory of controllability and filtration of stochastic systems. In theory of filtration he introduced and proved the Kalman filter for wideband noises and studied its invariance. He obtained first order necessary optimality conditions in stochastic systems, studied controllability and observability of a number of stochastic systems [174-178].

Another representative of this school is Nazim Mahmudov that defended PhD dissertation in 1984. He proved stochastic analogue of Pontryagin's maximum principle for finite-dimensional stochastic differential equations with controllable

diffusion, proved stochastic analogue of Pontryagin's maximum principle for infinite-dimensional stochastic differential equations, obtained new controllability conditions for determined and stochastic evolution equations in abstract spaces and stochastic analogue of discrete principle of maximum for Ito's discrete stochastic equations, obtained stronger necessary optimality conditions of first and second order for a wider class of optimal control problems in discrete systems [179-183].

In 1980's under the direction of the Academy of Sciences of Azerbaijan, a number of young specialists were sent to professional trip to major research centers as M.V. Lomonosov MSU, V.A. Steklov Institute of Mathematics of the Academy of Sciences, Computing Center of AS USSR, Institute of Mathematics and Mechanics of Ukrain, branch of AS USSR, Institute of Mathematics and Cybernetics of AS of Ukraine, Institute of Mathematics of AS of Byelorussia, etc. Further they became leading specialists in the field of mathematical theory of optimal control. Among them Abbas Azimov, Farhad Huseynov, Khaliq Huseynov, Elm khan Mahmudov successfully defended doctor's degree dissertations.

Abbas Azimov that defended in 1987 doctor's degree dissertation on the theme "Necessary conditions of optimality and duality in problems of optimizations on a cone" was engaged in investigation of theory of differential games under the guidance of professor M.S. Nikolskii [184,185].

Farhad Huseynov was engaged in mathematical economics and studied the issues connected with extension of many-dimensional variational problems [186,187-191].

Khaliq Huseynov defended doctor's degree dissertation in 1987. He studied the issues connected with control problems by the feedback principle, studied stable bridges in approachment problems and structure of stable sets in differential games [192, 193,194,195].

Elm khan Mahmudov that defended doctor's degree dissertation in 1992 on the theme "Optimization of discrete and differential inclusions with distributed parameters and duality", found relation between LAM and conjugate (nonlocal) mappings, proved duality theorems for convex multivalued mappings established in the terms of Hamilton's function [196,198], theorems on finiteness of the number of switchings for optimal control with multilateral differential inclusions [196], in conditions t_1 -transversality, in the terms of Euler-Lagrange's conjugate inclusions, derived new sufficient conditions of optimality for Bolts-type problem with ordinary differential inclusions and phase constraints [203], considered a new class of optimal control problems for higher order differential inclusions [202] and optimization of differential inclusions with a second order elliptic operator [197,201], studied optimization of elliptic, hyperbolic and parabolic-type discrete and differential inclusions and constructed theories of duality and equivalence [196,199, 200].

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