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ANALYSIS AND NUMERICAL SIMULATION OF FRACTIONAL BIOLOGICAL POPULATION MODEL WITH SINGULAR AND NON-SINGULAR KERNELS

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Abstract. This work investigates the fractional biological population model having carrying capacity. This fractional model is studied with singular and non-singular fractional derivatives. The Adams-Bashforth method is used to solve the predator-prey model with the non-local operator. Our numerical scheme is simply the combination of the fundamental theorem of integral calculus with Lagrange's interpolation. We use the fixed-point theorem to check the existence and uniqueness of this modified fractional model. We obtained different asymptotic behaviours for three different fractional derivatives that do not exist in the integer-order modelling. Finally, we present numerical results in tabulated and graphically form for distinct values of fractional order.

1. Introduction

In population ecology, the relationship between predator and prey significantly contributes. The predator is a species that primarily obtain food by killing and consuming other species known as prey. In our nature, we have several predator-prey pairs; examples of predator-prey pairs are lion and zebra, bear and fish, fox and rabbit etc. The survival of predator species is not possible in the absence of prey species.

Recently, many fractional-order biological population models have been developed to analyze the factor affecting the ecosystem. Freedman [12] examined the Gaussian system predator-prey model. Lotka [23] and Volterra [34] proposed the classical generalized predator-prey model. The fractional Lotka-Volterra model is analyzed by S. Das [9] using the homotopy perturbation method. C.A. Ibarra [16] examined the Bazykin predator-prey model with the ratio-dependent functional response and predator intraspecific interactions. E. Ahmed [2] analyzed fractional-order rabies and predator-prey models.

Fractional calculus (FC) is the generalized form of classical calculus. FC is a branch of applied science having enormous applications in engineering and science. During the past years, the evolution of fast and highly effective numerical

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techniques in fractional calculus drew the attention of researchers and scientists. The fractional models are more authentic and realistic in analyzing real-life problems than integer-order derivatives. The fractional derivative has significant applications in science and engineering, such as biological sciences, control theory, signal and systems, mechanics, chemical engineering, banking, fluid dynamic, plasma physics, neurophysiology, traffic, acoustics and many engineering sciences.

In FC, Caputo and Riemann-Liouville derivatives have been used at most. Still, they possess a singular power-law kernel, so we can't define the non-locality of real-life phenomena. That's why to illustrate nonlocal behaviour ABC [4] derivative with Mittag-Leffler kernel and CF [8] derivative with exponential decay kernel with non-singular kernels are presented. They are more authentic in illustrating real-life problems. These modern derivatives were used to investigate various fractional differential equations such as the fractional Ebola-Virus model [10], fractional Covid-19 model [33], El Nino-Southern Oscillation model [31], fractional immunogenetic tumour model [13], fractional nutrient-phytoplankton-zooplankton model [14], fractional Keller-Segel model [5], SIR epidemic model [30], fractional burgers equation [35], Spatiotemporal patterns in the Belousov-Zhabotinskii reaction system [25], Optimal control [3], Chickenpox disease model [28], fractional Cahn-Allen model [27], fractional banking model [11], fractional HIV epidemic model [20], fractional Tricomi equation [19], Nabla fractional boundary value problem [18], Hadamard fractional differential equations [1] etc.

This work's main objective is to study fractional biological population model with 3-step ABM via Caputo, CF, and ABC derivatives. This predator-prey model is related to the species' population, so the variables and parameters used are non-negative. The proposed biological population model is defined as follows:

$$\left.\begin{aligned} D_t^{\alpha}u(t) &= u(t)\left(a_1 - \tfrac{a_1 u(t)}{K_1}\right) - b_1 u(t)v(t), \\ D_t^{\alpha}v(t) &= v(t)\left(-a_2 + b_2 u(t)\right), \quad 0 < \alpha \le 1, \end{aligned}\right\} \tag{1.1}$$

with

$$u_0(t) = \lambda_1, v_0(t) = \lambda_2, \tag{1.2}$$

here *u(t)* and *v(t)* represent respectively prey and predator population density for time t, a_1 and a_2 denote, the intrinsic growth rate of prey and predator respectively, carrying capacity is denoted by K_1. b_1 and b_2 signify the competition coefficient of prey and predator, respectively. Here a_1, a_2, b_1, b_2 and K_1 are positive parameter.

In computational biology, fractional biological population model has many applications. Earlier, this fractional biological population model was analyzed by reproducing kernel Hilbert space method [7] and homotopy perturbation Sumudu transform approach [32] via Caputo derivative. The present work explores the proposed model with 3-step ABM with Caputo, CF, and ABC derivatives. Earlier fractional ABM was studied by Kumar [22] and Owolabi [24], Jena [17] analyzed coupled spring-mass system, Hamou [15] predicted COVID-19 with quarantine and isolation strategies with CF derivative.

Organization: In section 2 some basic definitions are defined. The existence and uniqueness of model's solutions is presented in section. Section 4 examines the fractional biological population model with 3-step ABM via singular and non-singular kernel derivatives. Simulation results are discussed in section 5, and the conclusion is drawn in section 6.

2. Preliminaries

This section includes basic definitions of Caputo, CF and ABC derivatives and integrals.
Definition 2.1. The Caputo derivative of $\phi \in \mathrm{C}_\beta, \beta \geq -1$ is given by [21, 26]:

$$ {}_0^C D_t^\alpha \phi(\mathrm{t}) = \left\{ \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-p)^{-\alpha} \phi'(p) dp, \text{ here } 0 < \alpha < 1 \right. . $$

Definition 2.2. Let $\phi \in H^1(c,d), d > \mathrm{c}$, and $\alpha \in [0,1]$, then the CF derivative of $\phi(t)$ is ginen by [6, 8]:

$$ {}^{CF}_0 D_t^\alpha \phi(t) = \frac{M(\alpha)}{1-\alpha} \int_0^t \phi'(p) \exp\left(-\frac{\alpha(t-p)}{1-\alpha}\right) dp, $$

and CF integral is given by:

$$ {}^{CF}_0 I_t^\alpha \phi(t) = \frac{1-\alpha}{M(\alpha)} \phi(t) + \frac{\alpha}{M(\alpha)} \int_0^t \phi(p) dp, t \geq 0, $$

here $M(\alpha)$ denote normalization function such that $M(0) = M(1) = 1$.
Definition 2.3. Let $\phi \in H^1(c,d), d > c, \alpha \in [0,1]$, then the ABC derivative is given by [4]:

$$ {}^{ABC}_0 D_t^\alpha \phi(t) = \frac{B(\alpha)}{1-\alpha} \int_0^t \phi'(p) E_\alpha \left[-\frac{\alpha}{1-\alpha} (t-p)^\alpha \right] dp, $$

and ABC integral is given by:

$$ {}^{ABC}_0 I_t^\alpha \phi(t) = \frac{1-\alpha}{B(\alpha)} \phi(t) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \int_0^t \phi(p)(t-p)^{\alpha-1} dp, t \geq 0, $$

here $B(\alpha)$ denote the normalization function such that $B(0) = B(1) = 1$.

3. Existence and uniqueness analysis

In this section, the existence and uniqueness of this fractional model is studied by using the fixed-point theorem. The model equation of 2-dimentional predator-prey system in term of ABC derivative is defined as:

$$ \left. \begin{array}{l} {}^{ABC}_0 D_t^\alpha u(t) = u(t) \left(a_1 - \frac{a_1 u(t)}{K_1} \right) - b_1 u(t) v(t), \\ {}^{ABC}_0 D_t^\alpha v(t) = v(t) \left(-a_2 + b_2 u(t) \right), 0 < \alpha \leq 1. \end{array} \right\} \tag{3.1} $$

Integrating Eq. (3.1) both sides by using definition of ABC integral, we have

$$\begin{aligned}
u(t)-u(0)=&\frac{(1-\alpha)}{B(\alpha)}\left(u(t)\left(a_1-\frac{a_1u(t)}{K_1}\right)-b_1u(t)v(t)\right)\\
&+\frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_0^t(t-p)^{\alpha-1}\left(u(p)\left(a_1-\frac{a_1u(p)}{K_1}\right)-b_1u(p)v(p)\right)dp,\\
v(t)-v(0)=&\frac{(1-\alpha)}{B(\alpha)}v(t)\left(-a_2+b_2u(t)\right)\\
&+\frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_0^t(t-p)^{\alpha-1}\left(v(p)\left(-a_2+b_2u(p)\right)\right)dp.
\end{aligned} \tag{3.2}$$

For simplicity, we define

$$\left.\begin{aligned}
F(t,u,v)&=u(t)\left(a_1-\tfrac{a_1u(t)}{K_1}\right)-b_1u(t)v(t),\\
G(t,u,v)&=v(t)\left(-a_2+b_2u(t)\right).
\end{aligned}\right\} \tag{3.3}$$

Now, we have to prove that the kernel $F(t,u,v)$ and $G(t,u,v)$ satisfies the Lipschitz condition.

Theorem 3.1. *The kernel F(t,u,v) and G(t,u,v) satisfy the Lipschitz condition and show contraction if:*

$$0<A_i\leq 1, i=1,2.$$

Proof. Let us suppose that $u(t)$ and $v(t)$ are bounded. So, if u and v have upper bound then, we have

$$\begin{aligned}
\|F(t,u,v)-F(t,u_1,v)\|=&\left\|u(t)\left(a_1-\frac{a_1u(t)}{K_1}\right)-b_1u(t)v(t)\right.\\
&\left.-u_1(t)\left(a_1-\frac{a_1u_1(t)}{K_1}\right)+b_1u_1(t)v(t)\right\|,
\end{aligned}$$

using property of norm, we have

$$\begin{aligned}
&\leq\|a_1(u(t)-u_1(t))\|+\left\|\frac{a_1}{K_1}(u(t)+u_1(t))(u(t)-u_1(t))\right\|\\
&\quad+\|(b_1v(t))(u(t)+u_1(t))\|,\\
&=\left\|\left(a_1+\frac{a_1}{K_1}(c_1+c_2)+b_1c_3\right)(u(t)-u_1(t))\right\|,\\
&\leq\left[a_1+\frac{a_1}{K_1}(c_1+c_2)+b_1c_3\right]\|u(t)-u_1(t)\|,
\end{aligned}$$

taking $A_1=\left[a_1+\frac{a_1}{K_1}(c_1+c_2)+b_1c_3\right]$, where u,u_1 and v are bounded functions such that $\|u\|\leq c_1, \|u_1\|\leq c_2$ and $\|v\|\leq c_3$, then

$$\|F(t,u,v)-F(t,u_1,v)\|\leq A_1\|u(t)-u_1(t)\|. \tag{3.4}$$

Thus, the kernel $F(t,u,v)$ satisfy Lipschitz condition. If in addition $0<A_1\leq 1$, then it is also a contraction. Similarly, we find $A_2=b_1c_1$ such that kernel

G(t,u,v) also satisfy Lipschitz condition as follow:

$$\|G(t,u,v)-G\left(t,u,v_1\right)\| \leq A_2\left\|v(t)-v_1(t)\right\|. \tag{3.5}$$

The kernel of model can be expressed as:

$$\begin{aligned}
u(t)&=u(0)+\frac{(1-\alpha)}{B(\alpha)}F(t,u,v)+\frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_0^t(t-p)^{\alpha-1}\ F(p,u,v)dp,\\
v(t)&=v(0)+\frac{(1-\alpha)}{B(\alpha)}G(t,u,v)+\frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_0^t(t-p)^{\alpha-1}G(p,u,v)dp.
\end{aligned}$$

We can write initial condition as:

$$u(0)=u_0, v(0)=v_0.$$

We construct the iterative formula as:

$$\begin{aligned}
u_n(t)&=u_0+\frac{(1-\alpha)}{B(\alpha)}F(t,u,v)+\frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_0^t(t-p)^{\alpha-1}\ F\left(p,u_{n-1},v\right)dp,\\
v_n(t)&=v_0+\frac{(1-\alpha)}{B(\alpha)}G(t,u,v)+\frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_0^t(t-p)^{\alpha-1}G\left(p,u,v_{n-1}\right)dp.
\end{aligned} \tag{3.6}$$

The succeeding terms difference is defined as follows:

$$\begin{aligned}
x_n(t)=u_n(t)-u_{n-1}(t)=&\frac{(1-\alpha)}{B(\alpha)}\left(F\left(t,u_{n-1},v\right)-F\left(t,u_{n-2},v\right)\right)+\frac{\alpha}{B(\alpha)\Gamma(\alpha)}\\
&\times\int_0^t(t-p)^{\alpha-1}\left(\ F\left(p,u_{n-1},v\right)-F\left(p,u_{n-2},v\right)\right)dp,\\
y_n(t)=v_n(t)-v_{n-1}(t)=&\frac{(1-\alpha)}{B(\alpha)}\left(G\left(t,u,v_{n-1}\right)-G\left(t,u,v_{n-2}\right)\right)+\frac{\alpha}{B(\alpha)\Gamma(\alpha)}\\
&\times\int_0^t(t-p)^{\alpha-1}\left(G\left(p,u,v_{n-1}\right)-G\left(p,u,v_{n-2}\right)\right)dp.
\end{aligned} \tag{3.7}$$

It can be written as follow:
$u_n(t)=\sum_{i=0}^n x_i(t)$, such that $x_0(t)=u_0$,
$v_n(t)=\sum_{i=0}^n y_i(t)$, such that $y_0(t)=v_0$.
Applying norm on first Eq. (3.7), we get

$$\begin{aligned}
\|x_n(t)\|=\|u_n(t)-u_{n-1}(t)\|=\|&\frac{(1-\alpha)}{B(\alpha)}\left(F\left(t,u_{n-1},v\right)-F\left(t,u_{n-2},v\right)\right)\\
&+\frac{\alpha}{B(\alpha)\Gamma(\alpha)}\\
&\times\int_0^t(t-p)^{\alpha-1}\left(\ F\left(p,u_{n-1},v\right)-F\left(p,u_{n-2},v\right)\right)dp\|.
\end{aligned} \tag{3.8}$$

Using norm's property on Eq. (3.8), we have

$$\begin{aligned}
\|x_n(t)\|\leq&\left\|\frac{(1-\alpha)}{B(\alpha)}\left(F\left(t,u_{n-1},v\right)-F\left(t,u_{n-2},v\right)\right)\right\|\\
&+\left\|\frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_0^t(t-p)^{\alpha-1}\left(\ F\left(p,u_{n-1},v\right)-F\left(p,u_{n-2},v\right)\right)dp\right\|,
\end{aligned}$$

using Eq. (3.4) in above inequality, we get

$$\begin{aligned}\|x_n(t)\| \leq & \frac{(1-\alpha)}{B(\alpha)} A_1 \|u_{n-1}(t) - u_{n-2}(t)\| \\ & + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} A_1 \int_0^t (t-p)^{\alpha-1} \|u_{n-1}(t) - u_{n-2}(t)\| \, dp, \\ \|x_n(t)\| \leq & \frac{(1-\alpha)}{B(\alpha)} A_1 \|x_{n-1}\| + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} A_1 \int_0^t (t-p)^{\alpha-1} \|x_{n-1}\| \, dp,\end{aligned}$$

similarly, we have the following result

$$\|y_n(t)\| \leq \frac{(1-\alpha)}{B(\alpha)} A_2 \|y_{n-1}\| + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} A_2 \int_0^t (t-p)^{\alpha-1} \|y_{n-1}\| \, dp. \tag{3.9}$$

Theorem 3.2. *The solutions of system (3.1) exists if there* $\exists\ t_0$ *which satisfy the following condition*

$$\left[\frac{1-\alpha}{B(\alpha)} A_i + \frac{\alpha}{B(\alpha)\Gamma(\alpha+1)} A_i t_0^\alpha\right] < 1, \text{ where } i = 1, 2. \tag{3.10}$$

Proof. Since u(t) and v(t) are bounded, kernel F(t,u,v) and G(t,u,v) hold the Lipschitz condition and using Eq. (3.9) and recursive method, we get

$$\|x_n(t)\| \leq \left[\tfrac{1-\alpha}{B(\alpha)} A_1 + \tfrac{\alpha}{B(\alpha)\Gamma(\alpha+1)} A_1 {t_0}^\alpha\right]^n \|u(0)\|,$$

$$\|y_n(t)\| \leq \left[\tfrac{1-\alpha}{B(\alpha)} A_2 + \tfrac{\alpha}{B(\alpha)\Gamma(\alpha+1)} A_2 {t_0}^\alpha\right]^n \|v(0)\|.$$

Therefore, the function $u_n(t) = \sum_{i=0}^n x_n(t)$ and $v_n(t) = \sum_{i=0}^n y_n(t)$ exist and smooth. Next, we have to prove that the above functions are the solutions of the fractional model.
Let

$$\begin{aligned} u(t) - u(0) &= u_n(t) - \phi_n(t), \\ v(t) - v(0) &= v_n(t) - \Psi_n(t). \end{aligned}$$

Therefore, we have

$$\begin{aligned}\|\phi_n(t)\| = & \left\| \frac{1-\alpha}{B(\alpha)} \{F(t,u,v) - F(t, u_{n-1}, v)\} \right. \\ & \left. + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \int_0^t (t-p)^{\alpha-1} \{\ F(p,u,v) - F(p, u_{n-1}, v)\}\, dp \right\|, \\ & \leq \frac{1-\alpha}{B(\alpha)} A_1 \|u(t) - u_{n-1}(t)\| + \frac{\alpha}{B(\alpha)\Gamma(\alpha+1)} A_1 t^\alpha \|u(t) - u_{n-1}(t)\|.\end{aligned}$$

Following the similar steps, we have

$$\|\phi_n(t)\| \leq \left[\frac{1-\alpha}{B(\alpha)} + \frac{\alpha}{B(\alpha)\Gamma(\alpha+1)} t^\alpha\right]^{n+1} A_1^{n+1} M.$$

Then at $t = t_0$, we have

$$\|\phi_n(t)\| \leq \left[\frac{1-\alpha}{B(\alpha)} + \frac{\alpha}{B(\alpha)\Gamma(\alpha+1)} t_0^\alpha\right]^{n+1} A_1^{n+1} M.$$

Now, $\|\phi_n(t)\| \to 0$ as $n \to \infty$. Similarly, we can show that $\|\Psi_n(t)\| \to 0$.
Hence, we show that the solutions exist.

Theorem 3.3. *The predator-prey model (3.1) has a unique system of solution if the condition below holds*

$$\left[1 - \frac{(1-\alpha)}{B(\alpha)}A_i - \frac{\alpha}{B(\alpha)\Gamma(\alpha+1)}A_i t^\alpha\right] > 0, \text{ where } i = 1, 2. \tag{3.11}$$

Proof. Let $u^*(t)$ and $v^*(t)$ be another set of solutions for this fractional biological population model. Now, taking

$$\begin{aligned} u(t) - u^*(t) =& \frac{(1-\alpha)}{B(\alpha)}\left(F(t,u,v) - F\left(t,u^*,v\right)\right) \\ &+ \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_0^t (t-p)^{\alpha-1}\left(\, F(p,u,v) - F\left(p,u^*,v\right)\right)dp, \end{aligned}$$

using norm

$$\begin{aligned} \|u(t) - u^*(t)\| \leq& \frac{(1-\alpha)}{B(\alpha)}\|F(t,u,v) - F(t,u^*,v)\| \\ &+ \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\left\|\int_0^t (t-p)^{\alpha-1}(\, F(p,u,v) - F(p,u^*,v))dp\right\|, \\ \leq& \frac{(1-\alpha)}{B(\alpha)}\|u(t) - u^*(t)\| + \frac{\alpha}{B(\alpha)\Gamma(\alpha+1)}t^\alpha \|u(t) - u^*(t)\|. \end{aligned}$$

This gives

$$\|u(t) - u^*(t)\|\left(1 - \frac{(1-\alpha)}{B(\alpha)}A_1 - \frac{\alpha}{B(\alpha)\Gamma(\alpha+1)}A_1 t^\alpha\right) \leq 0. \tag{3.12}$$

If the condition in Eq. (3.11) exists, then we have

$$\|u(t) - u^*(t)\| = 0. \tag{3.13}$$

Then, we get

$$u(t) = u^*(t),$$

following similar steps, we obtain

$$v(t) = v^*(t).$$

This verifies solution's uniqueness. Following similar steps, the proposed model's existence and uniqueness [17, 29] can easily be verified with Caputo and CF derivatives.

4. Numerical scheme

In this section, we examined the proposed model with the power-law kernel, Mittag-Leffler kernel and exponential decay kernel by using the 3-step ABM.

4.1. 3-step ABM via Caputo derivative.

We defined the proposed model in terms of Caputo derivative as:

$$\left.\begin{aligned} {}_0^C D_t^{\alpha} u(t) &= F(t,u,v), \\ {}_0^C D_t^{\alpha} v(t) &= G(t,u,v). \end{aligned}\right\} \tag{4.1}$$

Where

$$\left.\begin{aligned} F(t,u,v) &= u(t)\left(a_1 - \tfrac{a_1 u(t)}{K_1}\right) - b_1 u(t) v(t), \\ G(t,u,v) &= v(t)\left(-a_2 + b_2 u(t)\right). \end{aligned}\right\} \tag{4.2}$$

Integration 1^{st} equation of (4.1) in Caputo sense, we have

$$u(t) - u(0) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} F(s,u(s),v(s)) ds. \tag{4.3}$$

Using h as step size we discretizing the $[0,t]$ and have a sequence $t_0 = 0$, $t_{n+1} = t_0 + (n+1)h, n = 0,1,2,\ldots,q-1$, and $t_q = t$.
Set $t = t_{n+1}$ in Eq. (4.3), we get

$$u\left(t_{n+1}\right) - u(0) = \frac{1}{\Gamma(\alpha)} \int_0^{t_{n+1}} \left(t_{n+1}-s\right)^{\alpha-1} F(s,u(s),v(s)) ds. \tag{4.4}$$

By using above sequence Eq. (4.4) can be written as:

$$u\left(t_{n+1}\right) - u(0) = \frac{1}{\Gamma(\alpha)} \sum_{p=0}^{n} \int_{t_p}^{t_{p+1}} \left(t_{n+1}-s\right)^{\alpha-1} F(s,u(s),v(s)) ds. \tag{4.5}$$

In Eq. (4.5) we approximate the function $F(t,u,v)$ on the interval $[t_p, t_{p+1}]$ through the Lagrange polynomial as follows:

$$\begin{aligned} u\left(t_{n+1}\right) =& u(0) + \frac{1}{\Gamma(\alpha)} \sum_{p=2}^{n} \int_{t_p}^{t_{p+1}} \Bigg[\frac{F\left(t_p, u\left(t_p\right), v\left(t_p\right)\right)}{2\ h^2} \\ & \times \left(\ s - t_{p-1}\right)\left(s - t_{p-2}\right)\left(t_{n+1}-s\right)^{\alpha-1}\ ds \\ & - \int_{t_p}^{t_{p+1}} \frac{F\left(t_{p-1}, u\left(t_{p-1}\right), v\left(t_{p-1}\right)\right)}{h^2} \left(\ s - t_p\right)\left(s - t_{p-2}\right)\left(t_{n+1}-s\right)^{\alpha-1}\ ds \\ & + \int_{t_p}^{t_{p+1}} \frac{F\left(t_{p-2}, u\left(t_{p-2}\right), v\left(t_{p-2}\right)\right)}{2\ h^2} \left(\ s - t_p\right)\left(s - t_{p-1}\right)\left(t_{n+1}-s\right)^{\alpha-1}\ ds\Bigg]. \end{aligned} \tag{4.6}$$

Simplifying equation (4.6), we get

$$\begin{aligned} u(t_{n+1}) = u(0) + \frac{h^{\alpha}}{2\Gamma(\alpha+3)} \sum_{p=2}^{n} \Bigg[& F\big(t_p, u(t_p), v(t_p)\big)\big[(n+1-p)^{\alpha}\{2(n-p)^2 + (n \\ & - p)(3\alpha+10) + 2\alpha^2 + 9\alpha + 12\} - (n-p)^{\alpha}\{2(n-p)^2 + (n-p)(5\alpha+10) \\ & + 6\alpha^2 + 18\alpha + 12\}\big] - 2F\big(t_{p-1}, u(t_{p-1}), v(t_{p-1})\big)\big[(n+1-p)^{\alpha}\{2(n-p)^2 \\ & + (n-p)(2\alpha+8) + 2\alpha + 6\} - (n-p)^{\alpha}\{2(n-p)^2 + (n-p)(4\alpha+8) + 3\alpha^2 \\ & + 9\alpha + 6\}\big] + F\big(t_{p-2}, u(t_{p-2}), v(t_{p-2})\big)\big[(n+1-p)^{\alpha}\{2(n-p)^2 + (n-p)(\alpha \\ & + 6) + \alpha + 4\} - (n-p)^{\alpha}\{2(n-p)^2 + (n-p)(3\alpha+6) + 2\alpha^2 + 6\alpha + 4\}\big]\Bigg]. \end{aligned} \tag{4.7}$$

Similarly, for the 2^{nd} equation of (4.1), we can write

$$v(t_{n+1}) = v(0) + \frac{h^\alpha}{2\Gamma(\alpha+3)} \sum_{p=2}^{n} \Big[G\big(t_p, u(t_p), v(t_p)\big) \big[(n+1-p)^\alpha \{2(n-p)^2 + (n - p)(3\alpha+10) + 2\alpha^2 + 9\alpha + 12\} - (n-p)^\alpha \{2(n-p)^2 + (n-p)(5\alpha+10) + 6\alpha^2 + 18\alpha + 12\}\big] - 2G\big(t_{p-1}, u(t_{p-1}), v(t_{p-1})\big) \big[(n+1-p)^\alpha \{2(n-p)^2 + (n-p)(2\alpha+8) + 2\alpha + 6\} - (n-p)^\alpha \{2(n-p)^2 + (n-p)(4\alpha+8) + 3\alpha^2 + 9\alpha + 6\}\big] + G\big(t_{p-2}, u(t_{p-2}), v(t_{p-2})\big) \big[(n+1-p)^\alpha \{2(n-p)^2 + (n-p)(\alpha + 6) + \alpha + 4\} - (n-p)^\alpha \{2(n-p)^2 + (n-p)(3\alpha+6) + 2\alpha^2 + 6\alpha + 4\}\big] \Big]. \tag{4.8}$$

4.2. 3-step ABM via CF derivative.
We defined the proposed model in terms of CF derivative as:

$$\left. \begin{aligned} {}^{CF}_{0}D^{\alpha}_{t} u(t) &= F(t,u,v), \\ {}^{CF}_{0}D^{\alpha}_{t} v(t) &= G(t,u,v). \end{aligned} \right\} \tag{4.9}$$

Integration 1^{st} equation of (4.9) in CF sense, we get

$$u(t) - u(0) = \frac{1-\alpha}{M(\alpha)} F(t, u(t), v(t)) + \frac{\alpha}{M(\alpha)} \int_0^t F(s, u(s), v(s)) ds. \tag{4.10}$$

Using h as step size we discretizing the $[0,t]$ and have the sequence $t_0 = 0$, $t_{n+1} = t_0 + (n+1)h, n = 0, 1, 2, \ldots, q-1$, and $t_q = t$.
Set $t = t_{n+1}$ in Eq. (4.10), it follows that

$$u\left(t_{n+1}\right) - u(0) = \tfrac{1-\alpha}{M(\alpha)} F\left(t_n, u\left(t_n\right), v\left(t_n\right)\right) + \tfrac{\alpha}{M(\alpha)} \int_0^{t_{n+1}} F(s, u(s), v(s)) ds. \tag{4.11}$$

Set $t = t_{n+1}$ in Eq. (4.10), it follows that

$$u\left(t_n\right) - u(0) = \tfrac{1-\alpha}{M(\alpha)} F\left(t_{n-1}, u\left(t_{n-1}\right), v\left(t_{n-1}\right)\right) + \tfrac{\alpha}{M(\alpha)} \int_0^{t_n} F(s, u(s), v(s)) ds. \tag{4.12}$$

From (4.11) and (4.12), we get

$$\begin{aligned} u\left(t_{n+1}\right) - u\left(t_n\right) =& \frac{1-\alpha}{M(\alpha)} \left[F\left(t_n, u\left(t_n\right), v\left(t_n\right)\right) - F\left(t_{n-1}, u\left(t_{n-1}\right), v\left(t_{n-1}\right)\right)\right] \\ &+ \frac{\alpha}{M(\alpha)} \int_{t_n}^{t_{n+1}} F(s, u(s), v(s)) ds. \end{aligned} \tag{4.13}$$

Now, we approximate the function $F(t,u,v)$ on the interval $[t_n, t_{n+1}]$ through the Lagrange polynomial as follows:

$$\begin{aligned} F(t, u(t), v(t)) \cong& \frac{F\left(t_n, u\left(t_n\right), v\left(t_n\right)\right)}{\left(t_n - t_{n-1}\right)\left(t_n - t_{n-2}\right)} \left(t - t_{n-1}\right)\left(t - t_{n-2}\right) \\ &+ \frac{F\left(t_{n-1}, u\left(t_{n-1}\right), v\left(t_{n-1}\right)\right)}{\left(t_{n-1} - t_n\right)\left(t_{n-1} - t_{n-2}\right)} \left(t - t_n\right)\left(t - t_{n-2}\right) \\ &+ \frac{F\left(t_{n-2}, u\left(t_{n-2}\right), v\left(t_{n-2}\right)\right)}{\left(t_{n-2} - t_n\right)\left(t_{n-2} - t_{n-1}\right)} \left(t - t_n\right)\left(t - t_{n-1}\right). \end{aligned} \tag{4.14}$$

Using Eq. (4.14) in the integral term of Eq. (4.13), we get

$$\begin{aligned}\int_{t_n}^{t_{n+1}} F(s,u(s),v(s))ds = \int_{t_n}^{t_{n+1}} \Bigg[&\frac{F\left(t_n u\left(t_n\right), v\left(t_n\right)\right)}{\left(t_n-t_{n-1}\right)\left(t_n-t_{n-2}\right)}\left(t-t_{n-1}\right)\left(t-t_{n-2}\right) \\ &+\frac{F\left(t_{n-1}, u\left(t_{n-1}\right), v\left(t_{n-1}\right)\right)}{\left(t_{n-1}-t_n\right)\left(t_{n-1}-t_{n-2}\right)}\left(t-t_n\right)\left(t-t_{n-2}\right) \\ &+\frac{F\left(t_{n-2}, u\left(t_{n-2}\right), v\left(t_{n-2}\right)\right)}{\left(t_{n-2}-t_n\right)\left(t_{n-2}-t_{n-1}\right)}\left(t-t_n\right)\left(t-t_{n-1}\right)\Bigg].\end{aligned} \tag{4.15}$$

Simplifying Eq. (4.15), we get

$$\begin{aligned}\int_{t_n}^{t_{n+1}} F(s,u(s),v(s))ds =& h\left[\frac{23}{12}F\left(t_n, u\left(t_n\right), v\left(t_n\right)\right)-\frac{4}{3}F\left(t_{n-1}, u\left(t_{n-1}\right), v\left(t_{n-1}\right)\right)\right. \\ &\left.+\frac{5}{12}F\left(t_{n-2}, u\left(t_{n-2}\right), v\left(t_{n-2}\right)\right)\right].\end{aligned} \tag{4.16}$$

Using Eq. (4.16) in Eq. (4.13), we obtain

$$\begin{aligned}u\left(t_{n+1}\right)=u\left(t_n\right)&+\frac{1}{M(\alpha)}\left[(1-\alpha)+\frac{23}{12}h\alpha\right]F\left(t_n, u\left(t_n\right), v\left(t_n\right)\right) \\ &-\frac{1}{M(\alpha)}\left[(1-\alpha)+\frac{4}{3}h\alpha\right]F\left(t_{n-1}, u\left(t_{n-1}\right), v\left(t_{n-1}\right)\right) \\ &+\frac{5h\alpha}{12M(\alpha)}F\left(t_{n-2}, u\left(t_{n-2}\right), v\left(t_{n-2}\right)\right).\end{aligned} \tag{4.17}$$

Similarly, for the 2^{nd} equation of (4.9), we have

$$\begin{aligned}v\left(t_{n+1}\right)=v\left(t_n\right)&+\frac{1}{M(\alpha)}\left[(1-\alpha)+\frac{23}{12}h\alpha\right]G\left(t_n, u\left(t_n\right), v\left(t_n\right)\right) \\ &-\frac{1}{M(\alpha)}\left[(1-\alpha)+\frac{4}{3}h\alpha\right]G\left(t_{n-1}, u\left(t_{n-1}\right), v\left(t_{n-1}\right)\right) \\ &+\frac{5h\alpha}{12M(\alpha)}G\left(t_{n-2}, u\left(t_{n-2}\right), v\left(t_{n-2}\right)\right).\end{aligned} \tag{4.18}$$

4.3. 3-step ABM via ABC derivative.

We defined the proposed model in terms of ABC derivative as:

$$\left.\begin{aligned}{}^{ABC}_{\ \ 0}D_t^{\alpha}u(t)=F(t,u,v),\\ {}^{ABC}_{\ \ 0}D_t^{\alpha}v(t)=G(t,u,v).\end{aligned}\right\} \tag{4.19}$$

Integration 1^{st} equation of (4.19) in ABC sense, we get

$$u(t)-u(0)=\frac{1-\alpha}{B(\alpha)}F(t,u(t),v(t)+\frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}F(s,u(s),v(s))ds. \tag{4.20}$$

Using h as step size we discretizing the $[0,t]$ and have the sequence $t_0=0$, $t_{n+1}=t_0+(n+1)h, n=0,1,2,\dots,q-1$, and $t_q=t$.

Set $t=t_{n+1}$ in Eq. (4.20), it follows that

$$\begin{aligned} u\left(t_{n+1}\right)-u(0)= & \frac{1-\alpha}{B(\alpha)} F\left(t_n, u\left(t_n\right), v\left(t_n\right)\right) \\ & +\frac{\alpha}{B(\alpha) \Gamma(\alpha)} \int_0^{t_{n+1}}\left(t_{n+1}-s\right)^{\alpha-1} F(s, u(s), v(s)) d s . \end{aligned} \tag{4.21}$$

By using above sequence Eq. (4.21) can be written as:

$$\begin{aligned} u\left(t_{n+1}\right)-u(0)= & \frac{1-\alpha}{B(\alpha)} F\left(t_n, u\left(t_n\right), v\left(t_n\right)\right) \\ & +\frac{\alpha}{B(\alpha) \Gamma(\alpha)} \sum_{p=0}^n \int_{t_p}^{t_{p+1}}\left(t_{n+1}-s\right)^{\alpha-1} F(s, u(s), v(s)) d s . \end{aligned} \tag{4.22}$$

In Eq. (4.22) we approximate the function $F(t,u,v)$ on the interval $[t_p,t_{p+1}]$ through the Lagrange polynomial as follows:

$$\begin{aligned} u\left(\mathrm{t}_{\mathrm{n}+1}\right)= & u(0)+\frac{1-\alpha}{B(\alpha)} F\left(t_n, u\left(t_n\right), v\left(t_n\right)\right) \\ & +\frac{\alpha}{B(\alpha) \Gamma(\alpha)} \sum_{p=2}^n \int_{t_p}^{t_{p+1}}\left[\frac{F\left(t_p, u\left(t_p\right), v\left(t_p\right)\right)}{2\ h^2}\right. \\ & \times\left(\ s-t_{p-1}\right)\left(s-t_{p-2}\right)\left(t_{n+1}-s\right)^{\alpha-1}\ d s \\ & -\int_{t_p}^{t_{p+1}} \frac{F\left(t_{p-1}, u\left(t_{p-1}\right), v\left(t_{p-1}\right)\right)}{h^2}\left(\ s-t_p\right)\left(s-t_{p-2}\right)\left(t_{\mathrm{n}+1}-s\right)^{\alpha-1}\ d s \\ & \left.+\int_{t_p}^{t_{p+1}} \frac{F\left(t_{p-2}, u\left(t_{p-2}\right), v\left(t_{p-2}\right)\right)}{2\ h^2}\left(\ s-t_p\right)\left(s-t_{p-1}\right)\left(t_{n+1}-s\right)^{\alpha-1}\ d s\right] . \end{aligned} \tag{4.23}$$

Simplifying Eq. (4.23), we get

$$\begin{aligned} u(t_{n+1})= & u(0)+\frac{1-\alpha}{B(\alpha)} F\big(t_n, u(t_n), v(t_n)\big)+\frac{\alpha h^\alpha}{2 B(\alpha) \Gamma(\alpha+3)} \sum_{p=2}^n\Big[F\big(t_p, u(t_p), v(t_p)\big) \\ & [(n+1-p)^\alpha\{2(n-p)^2+(n-p)(3 \alpha+10)+2 \alpha^2+9 \alpha+12\}-(n-p)^\alpha \\ & \{2(n-p)^2+(n-p)(5 \alpha+10)+6 \alpha^2+18 \alpha+12\}]-2 F\big(t_{p-1}, u(t_{p-1}), v(t_{p-1})\big) \\ & [(n+1-p)^\alpha\{2(n-p)^2+(n-p)(2 \alpha+8)+2 \alpha+6\}-(n-p)^\alpha\{2(n-p)^2 \\ & +(n-p)(4 \alpha+8)+3 \alpha^2+9 \alpha+6\}]+F\big(t_{p-2}, u(t_{p-2}), v(t_{p-2})\big)[(n+1 \\ & -p)^\alpha\{2(n-p)^2+(n-p)(\alpha+6)+\alpha+4\}-(n-p)^\alpha\{2(n-p)^2+(n \\ & -p)(3 \alpha+6)+2 \alpha^2+6 \alpha+4\}]\Big] . \end{aligned} \tag{4.24}$$

Similarly, for the 2^{nd} equation of (4.19), we can write

$$\begin{aligned} v(t_{n+1}) = v(0) + \frac{1-\alpha}{B(\alpha)} G\big(t_n, u(t_n), v(t_n)\big) + \frac{\alpha h^\alpha}{B(\alpha) 2\Gamma(\alpha+3)} \sum_{p=2}^{n} \Big[G\big(t_p, u(t_p), v(t_p)\big) \\ [(n+1-p)^\alpha \{2(n-p)^2 + (n-p)(3\alpha+10) + 2\alpha^2 + 9\alpha + 12\} - (n-p)^\alpha \\ \{2(n-p)^2 + (n-p)(5\alpha+10) + 6\alpha^2 + 18\alpha + 12\}] - 2G\big(t_{p-1}, u(t_{p-1}), v(t_{p-1})\big) \\ [(n+1-p)^\alpha \{2(n-p)^2 + (n-p)(2\alpha+8) + 2\alpha + 6\} - (n-p)^\alpha \{2(n-p)^2 \\ + (n-p)(4\alpha+8) + 3\alpha^2 + 9\alpha + 6\}] + G\big(t_{p-2}, u(t_{p-2}), v(t_{p-2})\big) [(n+1 \\ -p)^\alpha \{2(n-p)^2 + (n-p)(\alpha+6) + \alpha + 4\} - (n-p)^\alpha \{2(n-p)^2 + (n \\ -p)(3\alpha+6) + 2\alpha^2 + 6\alpha + 4\}] \Big]. \end{aligned} \tag{4.25}$$

Table 1. Comparison results of fractional two-dimensional predator-prey system at $\alpha = 0.7$ defined in described in Caputo, CF and ABC sense.

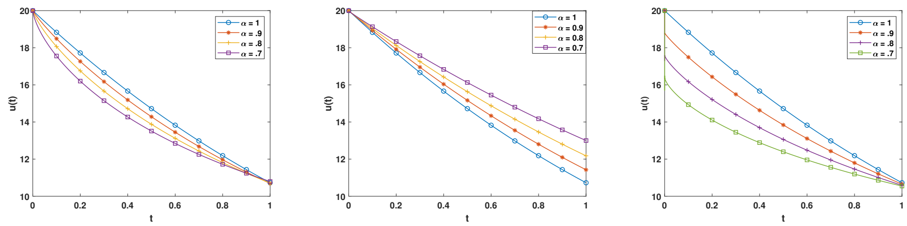
t	u^{Caputo}	u^{CF}	u^{ABC}	v^{Caputo}	v^{CF}	v^{ABC}
0	20.0000	20.0000	20.0000	15.0000	15.0000	15.0000
0.2	16.2020	18.3363	14.1058	15.6836	15.2903	15.9845
0.4	14.2680	16.8342	12.8869	16.0031	15.5428	16.1288
0.6	12.8458	15.4491	11.9564	16.2123	15.7643	16.2607
0.8	11.7204	14.1736	11.1943	16.3629	15.9582	16.3418
1	10.7956	13.0004	10.5477	16.4719	16.1256	16.4015

Table 2. Comparison results of fractional two-dimensional predator-prey system at $\alpha = 0.8$ defined in described in Caputo, CF and ABC sense.

t	u^{Caputo}	u^{CF}	u^{ABC}	v^{Caputo}	v^{CF}	v^{ABC}
0	20.0000	20.0000	20.0000	15.0000	15.0000	15.0000
0.2	16.7550	18.1270	15.2133	15.5903	15.3336	15.8295
0.4	14.7169	16.4263	13.6950	15.9358	15.6247	16.0602
0.6	13.1208	14.8744	12.4803	16.1860	15.8778	16.2288
0.8	11.8119	13.4614	11.4625	16.3730	16.0954	16.3562
1	10.7128	12.1772	10.5893	16.5132	16.2798	16.4530

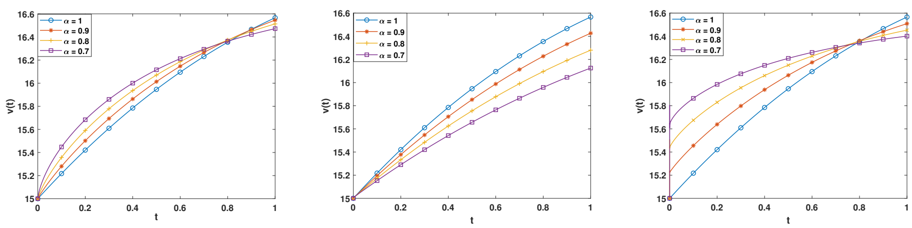
Table 3. Comparison results of fractional two-dimensional predator-prey system at $\alpha = 0.9$ defined in described in Caputo, CF and ABC sense.

t	u^{Caputo}	u^{CF}	u^{ABC}	v^{Caputo}	v^{CF}	v^{ABC}
0	20.0000	20.0000	20.0000	15.0000	15.0000	15.0000
0.2	17.2639	17.9201	16.4332	15.5018	15.3773	15.6388
0.4	15.1877	16.0374	14.6301	15.8629	15.7057	15.9384
0.6	13.4501	14.3358	13.1080	16.1467	15.9881	16.1745
0.8	11.9665	12.8027	11.7958	16.3707	16.2273	16.3618
1	10.6876	11.4254	10.6528	16.5458	16.4264	16.5091



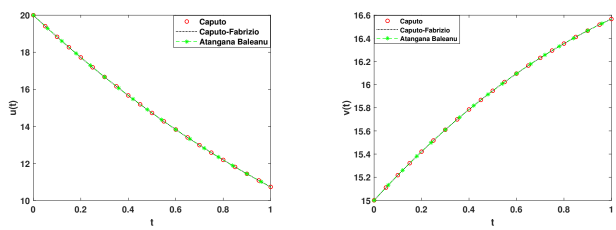
(A) Caputo derivative (B) CF derivative (C) ABC derivative

FIGURE 1. Graphical behaviour of $u(t)$ with t for $\alpha = 0.7, 0.8, 0.9, 1$ via (A) Caputo (B) CF (C) ABC derivative



(A) Caputo derivative (B) CF derivative (C) ABC derivative

FIGURE 2. Graphical behaviour of $v(t)$ with t for $\alpha = 0.7, 0.8, 0.9, 1$ via (A) Caputo (B) CF (C) ABC derivative



(A) (B)

FIGURE 3. Comparison results of (A) $u(t)$ with t (B) $v(t)$ with t at α= 1 via Caputo, CF and ABC derivative.

Table 4. Comparison results of fractional two-dimensional predator-prey system at $\alpha = 1$ defined in described in Caputo, CF and ABC sense.

t	u^{Caputo}	u^{CF}	u^{ABC}	v^{Caputo}	v^{CF}	v^{ABC}
0	20.0000	20.0000	20.0000	15.0000	15.0000	15.0000
0.2	17.7203	17.7182	17.7203	15.4206	15.4210	15.4206
0.4	15.6636	15.6644	15.6636	15.7849	15.7852	15.7849
0.6	13.8266	13.8249	13.8266	16.0950	16.0905	16.0950
0.8	12.1858	12.1842	12.1858	16.3543	16.3545	16.3543
1	10.7277	10.7264	10.7277	16.5664	16.5666	16.5664

5. Numerical simulation results

In this section, we present some graphs and tables to study the effect of fractional order on the population densities of prey (u) and predator (v). Here, we want to show the applicability, effectiveness and performance of ABM via Caputo, CF and ABC fractional derivatives for distinct values of fractional order $\alpha = 0.7, 0.8, 0.9, 1$. We take constant parameter as $a_1 = 0.05, a_2 = 0.05, b_1 = 0.04, b_2 = 0.01, K_1 = 20, \lambda_1 = 20, \lambda_2 = 15$ and step-size $h = 0.0001$. Fig. 1(A-C) represent the variation of $u(t)$ to t (time) for the distinct value of fractional parameter α with Caputo, CF and ABC fractional derivative. Fig. 1(A, C) show that there is a sharp decrease in density of prey species with decrease in value of α and with increase in value of time. Fig. 2(A-C) represent the variation of $v(t)$ to t (time) for distinct value of fractional parameter α with Caputo, CF and ABC fractional derivative. Fig. 2(A, C) show that there is sharp increase in density of predator species with decrease in value of α and with increase in value of time. But in case of CF derivative we get different behaviour than Caputo and ABC fractional derivative as shown in Fig. 1(B) and Fig. 2(B). Fig. 3(A) and 3(B) show the graphical comparison of result with Caputo, CF and ABC fractional derivative for $\alpha = 1$, which shows the efficiency and accuracy of the proposed numerical technique. Table 1-4 shows the comparison of numerical result with Caputo, CF and ABC fractional derivative at $\alpha = 0.7, 0.8, 0.9, 1$ respectively.

6. Conclusion

This work analyzed the nonlinear time-dependent predator-prey model with carrying capacity with singular and non-singular kernels to check the effects of the fractional order on the obtained solution by using 3-step ABM. We can easily use this numerical technique for nonlinear fractional problems. The simulation results are illustrated in tabular form and graphically. The proposed numerical technique is more reliable and efficient than the existing numerical approach. We obtained different asymptotic behaviour for different fractional order derivatives.

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