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WEIGHTED STATISTICAL CONVERGENCE OF ORDER α IN PROBABILITY

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Abstract. In this study, we introduce and examine the concepts of weighted statistical convergence of order α and weighted $\overline{[N_p]}-$summability of order α in probability. Also some relations between weighted statistical convergence of order α and weighted $\overline{[N_p]}-$summability of order α in probability are given.

1. Introduction

The idea of statistical convergence was introduced by Steinhaus [20] and Fast [10], and later reintroduced by Schoenberg [19] independently. Over the years and under different names statistical convergence has been discussed in the theory of Fourier analysis, Ergodic theory, Number theory, Measure theory, Trigonometric series, Turnpike theory and Banach spaces. Later on it was further investigated from the sequence space point of view and linked with summability theory by Bhunia et al. [2], Braha et al. [3], Colak [4], Connor [5], Fridy [11], Et et al. ([9],[21],[22]), Ghosal et al. ([6],[7],[8],[12],[13]), Isik et al. ([1],[14],[15],[16]) and many others. In recent years, generalizations of statistical convergence have appeared in the study of strong integral summability and the structure of ideals of bounded continuous functions on locally compact spaces. Statistical convergence and its generalizations are also connected with subsets of the Stone-Čech compactification of the natural numbers. Moreover, statistical convergence is closely related to the concept of convergence in probability.

Let $\mathbb{N}$ be the set of all natural numbers, $K \subseteq \mathbb{N}$ and $K(n) = \{k \leq n : k \in K\}$. The natural density of K is defined by $\delta(K) = \lim_n \frac{1}{n} |K(n)|$, if $\lim_n \frac{1}{n} |K(n)|$ exists. Here and from now, the vertical bars indicate the number of the elements in enclosed set.

The number sequence $x = (x_k)$ is said to be statistically convergent to L if for every $\varepsilon > 0$ the set $K(\varepsilon) = \{k \leq n : |x_k - L| \geq \varepsilon\}$ has natural density zero.

Weighted statistical convergence was first defined by Karakaya and Chishti [17] and the concept was modified by Mursaleen et al. [18]. Recently, Ghosal [12] revised the definition of weighted statistical convergence as follows.

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Let $p = (p_n)$ be a sequence of real numbers such that $\liminf p_n > 0$ and $P_n = p_1 + p_2 + p_3 + ... + p_n$ for all $n \in \mathbb{N}$. A sequence $x = (x_n)$ is said to be weighted statistically convergent to L if for every $\varepsilon > 0$

$$\lim_{n\to\infty} \frac{1}{P_n} \left|\left\{k \leq P_n \ : p_k \left|x_k - L\right| \geq \varepsilon\right\}\right| = 0.$$

In this case we write $S_{\overline{N}} - \lim x = L$. By $S_{\overline{N}}$, we denote the set of all weighted statistically convergent sequences.

Let X_n $(n \in \mathbb{N})$ be a sequence of random variables which is defined on a given event space S with respect to a given class of events Δ and a probability function $P : \Delta \to \mathbb{R}$, then we say that $X_1, X_2, X_3, ..., X_n$ ... is a sequence of random variables. A sequence of random variables is denoted by $\{X_n\}_{n\in\mathbb{N}}$.

A sequence $\{X_n\}_{n\in\mathbb{N}}$ is said to be statistically convergent in probability to a random variable X (where $X : S \to \mathbb{R}$) if for any $\delta, \varepsilon > 0$

$$\lim_{n\to\infty} \frac{1}{n} \left|\left\{k \leq n : P\left(\left|X_n - X\right| \geq \varepsilon\right) \geq \delta\right\}\right| = 0.$$

In this case we write $X_n \overset{PS}{\to} X$. The class of all sequences of random variables which are convergent in probability is denoted by PS.

A sequence of random variables $\{X_n\}$ is said to be bounded in probability, if for every $\delta > 0$ there exists $M > 0$ such that

$$P\left(\left|X_n\right| > M\right) < \delta, \text{ for all } n \in \mathbb{N},$$

that is,

$$\lim_{n\to\infty} P\left(\left|X_n\right| > M\right) = 0.$$

2. Main result

In the paper [12], again Ghosal defined the concepts of weighted statistical convergence of order α in probability and weighted $\overline{[N_p]}$−summability of order α in probability for $0 < \alpha \leq 1$. In this paper, we continue to examine other relations between weighted statistical convergence of order α in probability and weighted $\overline{[N_p]}$−summability of order α in probability for $0 < \alpha \leq 1$.

Let's start our study by giving two definitions.

Definition 2.1. [12] Let (S, Δ, P) be a probability space, $\{X_n\}_{n\in\mathbb{N}}$ be a sequence of random variables, $p = (p_n)$ be a sequence of real numbers such that $\liminf p_n > 0$, $P_n = p_1 + p_2 + p_3 + ... + p_n$ for all $n \in \mathbb{N}$ ($P_n = \sum_{k=1}^{n} p_k \to \infty$ as $n \to \infty$) and α be a fixed real number such that $\alpha \in (0, 1]$. A sequence of random variables $\{X_n\}$ is said to be weighted statistical convergence of order α in probability (weighted $PS^{\alpha}_{\overline{N_p}}$−statistically convergent) provided that for every $\varepsilon, \delta > 0$,

$$\lim_{n\to\infty} \frac{\left|\left\{k \leq P_n : p_k P\left(\left|X_k - X\right| \geq \varepsilon\right) \geq \delta\right\}\right|}{P_n^{\alpha}} = 0.$$

In this case we write $X_n \overset{PS^{\alpha}_{\overline{N_p}}}{\to} X$ $(n \to \infty)$. In case of $\alpha = 1$ we write $X_n \overset{PS_{\overline{N_p}}}{\to} X$ instead of $X_n \overset{PS^{\alpha}_{\overline{N_p}}}{\to} X$ and in case of $p_n = 1$ for all $n \in \mathbb{N}$ we write $X_n \overset{PS^{\alpha}_{\overline{N}}}{\to} X$ instead of $X_n \overset{PS^{\alpha}_{\overline{N_p}}}{\to} X$. In the special cases $\alpha = 1$ and $p_n = 1$ for all $n \in \mathbb{N}$, we write $X_n \overset{PS_{\overline{N}}}{\to} X$ instead of $X_n \overset{PS^{\alpha}_{\overline{N_p}}}{\to} X$. We denote the set of all weighted statistically convergent sequences of order α in probability by $PS^{\alpha}_{\overline{N_p}}$.

Definition 2.2. [12] Let (S, Δ, P) be a probability space, $\{X_n\}$ be a sequence of random variables, $p = (p_n)$ be a sequence of nonnegative real numbers such that $p_1 > 0$ $(P_n = \sum_{k=1}^{n} p_k \to \infty$ as $n \to \infty)$, α be a fixed real number such that $\alpha \in (0, 1]$ and $r > 0$ be a real number. A sequence of random variables $\{X_n\}$ is said to be weighted $\overline{[N_p]}(r)$ −summable of order α in probability provided that for every $\varepsilon > 0$,

$$\lim_{n\to\infty} \frac{1}{P_n^{\alpha}} \sum_{k=1}^{n} p_k \{P(|X_k - X| \geq \varepsilon)\}^r = 0.$$

In this case we write $X_n \overset{PW^{\alpha}_{\overline{N_p}}(r)}{\to} X$ $(n \to \infty)$ or $\lim PW^{\alpha}_{\overline{N_p}}(r) {-} P(|X_k - X| \geq \varepsilon) = 0$. In case of $r = 1$ we write $X_n \overset{PW^{\alpha}_{\overline{N_p}}}{\to} X$ instead of $X_n \overset{PW^{\alpha}_{\overline{N_p}}(r)}{\to} X$, In case of $\alpha = 1$ we write $X_n \overset{PW_{\overline{N_p}}(r)}{\to} X$ instead of $X_n \overset{PW^{\alpha}_{\overline{N_p}}(r)}{\to} X$ and in case of $p_n = 1$ for all $n \in \mathbb{N}$ we write $X_n \overset{PW^{\alpha}_{\overline{N}}(r)}{\to} X$ instead of $X_n \overset{PW^{\alpha}_{\overline{N_p}}(r)}{\to} X$. In the special cases $\alpha = 1$ and $p_n = 1$ for all $n \in \mathbb{N}$, we write $X_n \overset{PW_{\overline{N}}(r)}{\to} X$ instead of $X_n \overset{PW^{\alpha}_{\overline{N_p}}(r)}{\to} X$. We denote the set of all weighted $\overline{[N_p]}(r)$ −summable sequences of order α in probability by $PW^{\alpha}_{\overline{N_p}}(r)$.

Throughout the paper, we assume that the sequences (p_n) and (q_n) of real numbers such that $\liminf p_n > 0$, $\liminf q_n > 0$ and $P_n = p_1 + p_2 + p_3 + ... + p_n$, $Q_n = q_1 + q_2 + q_3 + ... + q_n$ with $p_n \leq q_n$ for all $n \in \mathbb{N}$.

Theorem 2.1. *Let α and β be fixed real numbers such that $0 < \alpha \leq \beta \leq 1$. If*

$$\liminf \frac{P_n^{\alpha}}{Q_n^{\beta}} > 0 \tag{2.1}$$

then $X_n \overset{PS^{\beta}_{\overline{N_q}}}{\to} X$ implies $X_n \overset{PS^{\alpha}_{\overline{N_p}}}{\to} X$.

Proof. Let $X_n \overset{PS^{\beta}_{\overline{N_q}}}{\to} X$, then

$$\lim_{n\to\infty} \frac{1}{Q_n^{\beta}} |\{k \leq Q_n : q_k P(|X_k - X| \geq \varepsilon) \geq \delta\}| = 0.$$

For given $\varepsilon > 0$ we have the following inclusion since $p_n \leq q_n$ for all $n \in \mathbb{N}$

$$\begin{aligned}\{k \leq Q_n : q_k P(|X_k - X| \geq \varepsilon) \geq \delta\} &\supseteq \{k \leq P_n : q_k P(|X_k - X| \geq \varepsilon) \geq \delta\} \\ &\supseteq \{k \leq P_n : p_k P(|X_k - X| \geq \varepsilon) \geq \delta\}\end{aligned}$$

and so we have the following inequality

$$\begin{aligned} &\frac{1}{Q_n^\beta}\left|\left\{k\leq Q_n : q_k P\left(|X_k-X|\geq\varepsilon\right)\geq\delta\right\}\right| \\ &\qquad\geq \tfrac{1}{Q_n^\beta}\left|\left\{k\leq P_n : p_k P\left(|X_k-X|\geq\varepsilon\right)\geq\delta\right\}\right| \\ &\qquad= \tfrac{P_n^\alpha}{Q_n^\beta}\tfrac{1}{P_n^\alpha}\left|\left\{k\leq P_n : p_k P\left(|X_k-X|\geq\varepsilon\right)\geq\delta\right\}\right|. \end{aligned}$$

Hence $X_n \overset{PS_{\overline{N_p}}^{\alpha}}{\rightarrow} X$. □

Corollary 2.1. *Let $\{X_n\}$ be a sequence of random variables, then*

(1) *If* $\liminf \dfrac{P_n^\alpha}{Q_n^\alpha} > 0$, *then* $X_n \overset{PS_{\overline{N_q}}^{\alpha}}{\rightarrow} X$ *implies* $X_n \overset{PS_{\overline{N_p}}^{\alpha}}{\rightarrow} X$ *for* $\alpha \in (0,1]$,
(2) *If* $\liminf \dfrac{P_n^\alpha}{Q_n} > 0$, *then* $X_n \overset{PS_{\overline{N_q}}}{\rightarrow} X$ *implies* $X_n \overset{PS_{\overline{N_p}}^{\alpha}}{\rightarrow} X$ *for* $\alpha \in (0,1]$,
(3) *If* $\liminf \dfrac{P_n}{Q_n} > 0$, *then* $X_n \overset{PS_{\overline{N_q}}}{\rightarrow} X$ *implies* $X_n \overset{PS_{\overline{N_p}}}{\rightarrow} X$.

Theorem 2.2. *Let α and β be fixed real numbers such that $0 < \alpha \leq \beta \leq 1$. If the condition* (2.1) *is satisfied, then* $X_n \overset{PW_{\overline{N_q}}^{\beta}(r)}{\rightarrow} X$ *implies* $X_n \overset{PW_{\overline{N_p}}^{\alpha}(r)}{\rightarrow} X$.

Proof. Proof similar to that of Theorem 2.1. □

Corollary 2.2. *Let $\{X_n\}$ be a sequence of random variables, then*

(1) *If* $\liminf \dfrac{P_n^\alpha}{Q_n^\alpha} > 0$, *then* $X_n \overset{PW_{\overline{N_q}}^{\alpha}}{\rightarrow} X$ *implies* $X_n \overset{PW_{\overline{N_p}}^{\alpha}}{\rightarrow} X$ *for* $\alpha \in (0,1]$,
(2) *If* $\liminf \dfrac{P_n^\alpha}{Q_n} > 0$, *then* $X_n \overset{PW_{\overline{N_q}}}{\rightarrow} X$ *implies* $X_n \overset{PW_{\overline{N_p}}^{\alpha}}{\rightarrow} X$ *for* $\alpha \in (0,1]$,
(3) *If* $\liminf \dfrac{P_n}{Q_n} > 0$, *then* $X_n \overset{PW_{\overline{N_q}}}{\rightarrow} X$ *implies* $X_n \overset{PW_{\overline{N_p}}}{\rightarrow} X$.

Theorem 2.3. *Let α and β are fixed real numbers such that $0 < \alpha \leq \beta \leq 1$, $\{X_n\}$ be a sequence of random variables. If the condition* (2.1) *is satisfied, then* $X_n \overset{PW_{\overline{N_q}}^{\beta}}{\rightarrow} X$ *implies* $X_n \overset{PS_{\overline{N_p}}^{\alpha}}{\rightarrow} X$.

Proof. Let $X_n \overset{PW_{\overline{N_q}}^{\beta}}{\rightarrow} X$. For any sequences $\{X_n\}$ of random variables and $\varepsilon, \delta > 0$, we have

$$\begin{aligned}
& \frac{1}{Q_n^{\beta}} \sum_{k=1}^{n} q_k P\left(|X_k - X| \geq \varepsilon\right) \\
= & \frac{1}{Q_n^{\beta}} \sum_{\substack{k=1 \\ p_k P(|X_k - X| \geq \varepsilon) \geq \delta}}^{n} q_k P\left(|X_k - X| \geq \varepsilon\right) \\
+ & \frac{1}{Q_n^{\beta}} \sum_{\substack{k=1 \\ p_k P(|X_k - X| \geq \varepsilon) < \delta}}^{n} q_k P\left(|X_k - X| \geq \varepsilon\right) \\
\geq & \frac{1}{Q_n^{\beta}} \sum_{\substack{k=1 \\ p_k P(|X_k - X| \geq \varepsilon) \geq \delta}}^{n} q_k P\left(|X_k - X| \geq \varepsilon\right) \\
\geq & \frac{1}{Q_n^{\beta}} \sum_{\substack{k=1 \\ p_k P(|X_k - X| \geq \varepsilon) \geq \delta}}^{[Q_n]} q_k P\left(|X_k - X| \geq \varepsilon\right) \\
\geq & \frac{1}{Q_n^{\beta}} \left|\left\{k \leq Q_n : q_k P\left(|X_k - X| \geq \varepsilon\right) \geq \delta\right\}\right| \delta \\
\geq & \frac{P_n^{\alpha}}{Q_n^{\beta}} \frac{1}{P_n^{\alpha}} \left|\left\{k \leq P_n : p_k P\left(|X_k - X| \geq \varepsilon\right) \geq \delta\right\}\right| \delta
\end{aligned}$$

Hence $X_n \overset{PS_{\overline{N_p}}^{\alpha}}{\rightarrow} X$. ☐

Corollary 2.3. *Let $\{X_n\}$ be a sequence of random variables, then*

1. *If* $\liminf \frac{P_n^{\alpha}}{Q_n^{\alpha}} > 0$, *then* $X_n \overset{PW_{\overline{N_q}}^{\alpha}}{\rightarrow} X$ *implies* $X_n \overset{PS_{\overline{N_p}}^{\alpha}}{\rightarrow} X$ *for* $\alpha \in (0,1]$,
2. *If* $\liminf \frac{P_n^{\alpha}}{Q_n} > 0$, *then* $X_n \overset{PW_{\overline{N_q}}}{\rightarrow} X$ *implies* $X_n \overset{PS_{\overline{N_p}}^{\alpha}}{\rightarrow} X$ *for* $\alpha \in (0,1]$,
3. *If* $\liminf \frac{P_n}{Q_n} > 0$, *then* $X_n \overset{PW_{\overline{N_q}}}{\rightarrow} X$ *implies* $X_n \overset{PS_{\overline{N_p}}}{\rightarrow} X$.

Theorem 2.4. *Let $p = (p_n)$ be a sequence of real numbers such that $\liminf p_n > 0$, α and β are fixed real numbers such that $0 < \alpha \leq \beta \leq 1$, and $\{X_n\}$ be a weighted $\overline{[N_p]}(r)-$summable sequence of order α to X. If $0 < r < 1$ and $0 < (p_k P(|X_k - X| \geq \varepsilon)) < 1$, then $\{X_n\}$ is weighted $PS_{\overline{N_p}}^{\beta}-$statistically convergent to X.*

Proof. Since $\{X_n\}$ weighted $\overline{[N_p]}(r)-$summable of order α to X, we have

$$\lim_{n\to\infty} \frac{1}{P_n^{\alpha}} \sum_{k=1}^{n} p_k \left\{P\left(|X_k - X| \geq \varepsilon\right)\right\}^r = 0$$

If $0 < r < 1$ and $0 < (p_k P(|X_k - X| \geq \varepsilon)) < 1$, then we can write

$$p_k \{P(|X_k - X| \geq \varepsilon)\}^r \geq p_k \{P(|X_k - X| \geq \varepsilon)\}.$$

So we have

$$\begin{aligned}\sum_{k=1}^{n} p_k \{P(|X_k - X| \geq \varepsilon)\}^r &\geq \sum_{k=1}^{n} p_k \{P(|X_k - X| \geq \varepsilon)\} \\ &\geq \sum_{\substack{k=1 \\ p_k P(|X_k - X| \geq \varepsilon) \geq \delta}}^{[P_n]} p_k \{P(|X_k - X| \geq \varepsilon)\} \\ &\geq |\{k \leq P_n : (p_k \{P(|X_k - X| \geq \varepsilon)\}) \geq \delta\}| \delta\end{aligned}$$

and so that

$$\begin{aligned}&\frac{1}{P_n^{\beta}} |\{k \leq P_n : (p_k \{P(|X_k - X| \geq \varepsilon)\}) \geq \delta\}| \delta \\ &\qquad \leq \tfrac{1}{P_n^{\alpha}} |\{k \leq P_n : (p_k \{P(|X_k - X| \geq \varepsilon)\}) \geq \delta\}| \delta \\ &\qquad \leq \tfrac{1}{P_n^{\alpha}} \sum_{k=1}^{n} p_k \{P(|X_k - X| \geq \varepsilon)\}^r \to 0.\end{aligned}$$

This means that $\{X_n\}$ is weighted $PS_{\overline{N_p}}^{\beta}-$statistically convergent to X. □

Theorem 2.5. *Let $p = (p_n)$ be a sequence of real numbers such that $\liminf p_n > 0$, α and β are fixed real numbers such that $0 < \alpha \leq \beta \leq 1$, and $\{X_n\}$ be a weighted $\overline{[N_p]}(r)-$summable sequence of order α to X. If $1 \leq r < \infty$ and $1 \leq (p_k P(|X_k - X| \geq \varepsilon)) < \infty$, then $\{X_n\}$ is weighted $PS_{\overline{N_p}}^{\beta}-$statistically convergent to X.*

Proof. Proof is similar to that of Theorem 2.4. □

Theorem 2.6. *Let $p = (p_n)$ be a sequence of real numbers such that $\liminf p_n > 0$, let $\{X_n\}$ be a weighted $PS_{\overline{N_p}}-$statistically convergent sequence to X and $(p_k P(|X_k - X| \geq \varepsilon)) \leq M$. If the following assertions hold, then $\{X_n\}$ is weighted $\overline{[N_p]}(r)-$summable sequence to X.*

(1) $0 < r < 1$ *and* $1 \leq M < \infty$,
(2) $1 \leq r < \infty$ *and* $0 \leq M < 1$.

Proof. Suppose that $\{X_n\}$ is a weighted $PS_{\overline{N_p}}^{\beta}-$statistically convergent sequence to X. Then for every $\varepsilon, \delta > 0$ we have $\delta_{\bar{N}}(K(\varepsilon)) = 0$, where

$$K(\varepsilon) = \{k \in \mathbb{N} : p_k P(|X_k - X| \geq \varepsilon) \geq \delta\}.$$

Write $K_{P_n}(\varepsilon) = \{k \le P_n : p_k \{P(|X_k - X| \ge \varepsilon)\}^r \ge \delta\}$. Since $p_k P(|X_k - X| \ge \varepsilon) \le M$ $(k = 1, 2, ...)$ we have

$$\begin{aligned}\frac{1}{P_n}\sum_{k=1}^{n} p_k \{P(|X_k - X| \ge \varepsilon)\}^r &= \frac{1}{P_n}\sum_{\substack{k=1\\ k\notin K_{P_n}(\varepsilon)}}^{n} p_k \{P(|X_k - X| \ge \varepsilon)\}^r \\ &\quad + \frac{1}{P_n}\sum_{\substack{k=1\\ k\in K_{P_n}(\varepsilon)}}^{n} p_k \{P(|X_k - X| \ge \varepsilon)\}^r \\ &\le \delta + M\frac{|K_{P_n}(\delta)|}{P_n} \to 0.\end{aligned}$$

Thus $\{X_n\}$ is weighted $\overline{[N_p]}(r)$ −summable sequence to X. □

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