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A STUDY ON INTUITIONISTIC FUZZY SOFT SUPRA TOPOLOGICAL SPACES

CIGDEM GUNDUZ ARAS

Abstract. In this paper, we introduce intuitionistic fuzzy soft supra topological space and then we define the notions of intuitionistic fuzzy soft supra closure, intuitionistic fuzzy soft supra interior. Later we investigate some of their important properties. We also consider the notions of fuzzy strongly soft supra connected space, fuzzy soft supra continuous mapping and intuitionistic fuzzy soft supra compact space. We investigate many properties, relations and characterizations of these notions.

1. Introduction

After the introduction of the concept of fuzzy sets by Zadeh [32], Chang [11] introduced the concept of fuzzy topological spaces. Later Kubiak [20] and Sostak [30] considered independently the concept of fuzzy topology. As a generalization of fuzzy sets; the concept of intuitionistic fuzzy sets was introduced by Atanassov [5]. Coker [12] and his colleagues introduced intuitionistic fuzzy topological spaces by using intuitionistic fuzzy sets. Much study has been done with these concepts.

Mashhour et al. [23] introduced the concepts of supra topological spaces, supra closed sets, supra open sets and supra continuous mapping. Later Elmonsef and Ramadan [2] introduced a fuzzy supra topological space and defined the concepts of fuzzy supra open sets, fuzzy supra closed sets and fuzzy supra continuity as a generalization of fuzzy continuity. Abbas [1] defined intuitionistic supra fuzzy topological space induced by an intuitionistic fuzzy bitopological space. Also he studied the relationship between intuitionistic supra fuzzy closure space and the intuitionistic supra fuzzy topological space induced by an intuitionistic fuzzy bitopological space. Turanlı [31] presented some fundamental results of intuitionistic fuzzy supra topological space and gave several connectedness concepts and a notion of compactness in these spaces.

Molodtsov [25] introduced the concept of soft set theory which is a completely new approach for modeling uncertainty. After Molodtsov's work, some different applications of soft sets and fuzzy(intuitionistic) soft sets in topology and algebra were studied in [3],[4],[9] ,[14],[15],[16],[18],[21],[22],[26],[27]. Up to the present, research on soft sets and fuzzy (intuitonistic) soft sets have been very active and many important results have been achieved in the theoretical aspect.

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Shabir and Naz [28] initiated the study of soft topological spaces and define some notions of soft topological spaces. As a generalized of soft topological spaces, S.A.El-Sheikh and A.M.Abd-El-Latif [13] introduced the notion of supra soft topological spaces by neglecting only the soft intersection condition.

The theoretical studies of soft topological spaces were studied in [6],[17],[10],[8], [19], [24],[29]. In these studies, the concept of a soft point is expressed using different approaches. In the present study, we use the concept of a soft point as given in [6].

In this paper, we introduce intuitionistic fuzzy soft supra topological space and then we define the notions of intuitionistic fuzzy soft supra closure, intuitionistic fuzzy soft supra interior. Later we investigate some of their important properties. We also consider the notions of fuzzy strongly soft supra connected space, fuzzy soft supra continuous mapping and intuitionistic fuzzy soft supra compactness. We examine many properties relations and characterizations of these notions.

2. Preliminaries

In this section, we present the basic definitions and results of intuitionistic fuzzy soft set theory which will be used in sequel.

Definition 2.1. [5] Let X be a non-empty set. An intuitionistic fuzzy set A in X ($IFS\left(X\right)$ for short) is an object having the form

$$A=\left\{\left\langle x,\mu_A\left(x\right),\lambda_A\left(x\right)\right\rangle : x\in X\right\}$$

where the functions $\mu_A,\lambda_A : X\rightarrow I=[0,1]$ define respectively, the degree of membership and the degree of non-membership of the element $x\in X$ to the set A, which is a subset of X, and for every element $x\in X$, $0\leq\mu_A\left(x\right)+\lambda_A\left(x\right)\leq 1$.

Definition 2.2. [25] Let X be an universal set, E be a non-empty set of parameters and $P(X)$ denote the power set of X. A pair (F,E) is called a soft set over X if F is a mapping of E into the set of all subsets of the set X,i.e., $F:E\rightarrow P(X)$.

Definition 2.3. [7] Let $IFS\left(X\right)$ be set of all intuitionistic fuzzy sets on universe set X.A pair of (F,G) is called an intuitionistic fuzzy soft set over X with parameter E, where F,G are mappings from E into $IFS\left(X\right)$. That is, for all $e\in E$, $\left(F,G\right)\left(e\right):X\rightarrow I$ is an intuitionistic fuzzy set on X.

Definition 2.4. [7] For two intuitionistic fuzzy soft sets (F_1,G_1) and (F_2,G_2) over a common universe X with parameter E , we say that (F_1,G_1) is an intuitionistic fuzzy soft subset of (F_2,G_2) and write $(F_1,G_1)\subset(F_2,G_2)$ if $F_1\left(e\right)\leq F_2\left(e\right), G_1\left(e\right)\geq G_2\left(e\right)$.

Definition 2.5. [7] Two intuitionistic fuzzy soft sets (F_1,G_1) and (F_2,G_2) over a common universe X with parameter E are said to be equal if $(F_1,G_1)\subset(F_2,G_2)$ and $(F_2,G_2)\subset(F_1,G_1)$.

Definition 2.6. [7] The union of two intuitonistic fuzzy soft sets (F_1,G_1) and (F_2,G_2) over a common universe X with parameter E is the intuitionistic fuzzy soft set (H,Q) and $(H,Q)\left(e\right)=\left(F_1\left(e\right)\vee F_2\left(e\right), G_1\left(e\right)\wedge G_2\left(e\right)\right)$ for all $e\in E$. It is written as $(F_1,G_1)\cup(F_2,G_2)=(H,Q)$.

Definition 2.7. [7] The intersection of two intuitonistic fuzzy soft sets (F_1, G_1) and (F_2, G_2) over a common universe X with parameter E is the intuitionistic fuzzy soft set (H, Q) and $(H, Q)(e) = (F_1(e) \wedge F_2(e), G_1(e) \vee G_2(e))$ for all $e \in E$. It is written as $(F_1, G_1) \cap (F_2, G_2) = (H, Q)$.

Definition 2.8. [7] An intuitonistic fuzzy soft set (F, G) over X with parameter E is said to be null intuitionistic fuzzy soft set and is denoted by $\widetilde{\Phi}$, if and only if for each $e \in E$, $(F, G)(e) = (\underline{0}, \underline{1})$, where $\underline{0}$ is the membership function of the null fuzzy set over X and $\underline{1}$ is the membership function of the absolute fuzzy set over X .

Definition 2.9. [7] An intuitonistic fuzzy soft set (F, G) over X with parameter E is said to be an absolute intuitionistic fuzzy soft set and is denoted by $\widetilde{1}$, if and only if for all $e \in E$, $(F, G)(e) = (\underline{1}, \underline{0})$.

Definition 2.10. [7] The complement of an intuitonistic fuzzy soft set (F, G) over X is denoted by $(F, G)'$ and is defined by $(F, G)'(e) = (G(e), F(e))$ for all $e \in E$.

Definition 2.11. [31] An intuitionistic fuzzy supra topological space (X, τ) is said to be fuzzy strongly supra connected if X has no non-zero intuitionistic fuzzy supra closed set A and B in X such that $\mu_A + \mu_B \leq 1$ and $\lambda_A + \lambda_B \geq 1$.

3. Intuitionistic fuzzy soft supra topology

Let $IFSS(X, E)$ be a family of all intuitionistic fuzzy soft sets over X with parameters in E.

Definition 3.1. A family τ of $IFSS's$ on X is called an intuitonistic fuzzy soft supra topology ($IFSST$ for short) on X if

(1) $\widetilde{\Phi} \in \tau$ and $\widetilde{1} \in \tau$
(2) τ is closed under arbitrary suprema.

Then we call the pair (X, τ, E) as an intuitionistic fuzzy soft supra topological spaces ($IFSSTS$ for short). Then each member of τ is called an intuitionistic fuzzy soft supra open set.

Definition 3.2. Let (X, τ, E) be an intuitionistic fuzzy soft supra topological space. An intuitonistic fuzzy soft set (F, G) over X is said to be an intuitionistic fuzzy soft supra closed set, if its complement $(F, G)'$ belongs to τ.

Let $\mathfrak{F}$ be a family of intuitionistic fuzzy soft supra closed sets.

Proposition 3.1. *Let (X, τ, E) be an intuitionistic fuzzy soft supra topological spaces. Then*

(1) $\widetilde{\Phi}$ and $\widetilde{1}$ are intuitionistic fuzzy soft supra closed sets,
(2) $\mathfrak{F}$ is closed under arbitrary infima.

Remark 3.1. Let $\mathfrak{F}$ be a family that meets above the proposition conditions. Then the complement of elements $\mathfrak{F}$ constitute an intuitionistic fuzzy soft supra topology.

Example 3.1. Let $X = \{x_1, x_2\}$ and $E = \{e_1, e_2\}$. The intuitionistic fuzzy soft sets $(F_i, G_i) : E \to IFS(X)$ on X are defined as follows:

$(F_1, G_1)(e_1) = \left\langle \left(x_1, \left(\frac{1}{5}, \frac{2}{3}\right)\right), \left(x_2, \left(\frac{1}{2}, \frac{1}{3}\right)\right) \right\rangle,$
$(F_1, G_1)(e_2) = \left\langle \left(x_1, \left(\frac{1}{4}, \frac{1}{4}\right)\right), (x_2, (1, 0)) \right\rangle,$
$(F_2, G_2)(e_1) = \left\langle \left(x_1, \left(\frac{1}{2}, \frac{1}{2}\right)\right), \left(x_2, \left(\frac{1}{4}, \frac{1}{5}\right)\right) \right\rangle,$
$(F_2, G_2)(e_2) = \left\langle \left(x_1, \left(\frac{1}{5}, \frac{1}{4}\right)\right), \left(x_2, \left(\frac{1}{2}, 0\right)\right) \right\rangle,$
$(F_3, G_3)(e_1) = \left\langle \left(x_1, \left(\frac{1}{2}, \frac{1}{2}\right)\right), \left(x_2, \left(\frac{1}{2}, \frac{1}{5}\right)\right) \right\rangle,$
$(F_3, G_3)(e_2) = \left\langle \left(x_1, \left(\frac{1}{4}, \frac{1}{4}\right)\right), (x_2, (1, 0)) \right\rangle.$

Then $\tau = \left\{\widetilde{\Phi}, \widetilde{1}, (F_1, G_1), (F_2, G_2), (F_3, G_3)\right\}$ is an intuitionistic fuzzy soft supra topology on X.

Example 3.2. Let us consider the Example 3.1. Then the complements of (F_i, G_i) on X,$1 \leq i \leq 3$, are defined as follows:
$(F_1, G_1)'(e_1) = \left\langle \left(x_1, \left(\frac{2}{3}, \frac{1}{5}\right)\right), \left(x_2, \left(\frac{1}{3}, \frac{1}{2}\right)\right) \right\rangle,$
$(F_1, G_1)'(e_2) = \left\langle \left(x_1, \left(\frac{1}{4}, \frac{1}{4}\right)\right), (x_2, (0, 1)) \right\rangle,$
$(F_2, G_2)'(e_1) = \left\langle \left(x_1, \left(\frac{1}{2}, \frac{1}{2}\right)\right), \left(x_2, \left(\frac{1}{5}, \frac{1}{4}\right)\right) \right\rangle,$
$(F_2, G_2)'(e_2) = \left\langle \left(x_1, \left(\frac{1}{4}, \frac{1}{5}\right)\right), \left(x_2, \left(0, \frac{1}{2}\right)\right) \right\rangle,$
$(F_3, G_3)'(e_1) = \left\langle \left(x_1, \left(\frac{1}{2}, \frac{1}{2}\right)\right), \left(x_2, \left(\frac{1}{5}, \frac{1}{2}\right)\right) \right\rangle,$
$(F_3, G_3)'(e_2) = \left\langle \left(x_1, \left(\frac{1}{4}, \frac{1}{4}\right)\right), (x_2, (0, 1)) \right\rangle.$

Hence $\mathfrak{F} = \left\{\widetilde{\Phi}, \widetilde{1}, (F_1, G_1)', (F_2, G_2)', (F_3, G_3)'\right\}$ is a family of an intuitionistic fuzzy soft supra closed sets.

Proposition 3.2. *Let (X, τ, E) be an intuitionistic fuzzy soft supra topological space over X.*

$$\tau_e = \{(F, G)(e) : (F, G) \in \tau\}$$

is an intuitionistic fuzzy supra topology on X for each $e \in E$.

Remark 3.2. An intuitionistic fuzzy soft supra topology is a very natural generalization of intuitionistic fuzzy supra topology.

Remark 3.3. Every intuitonistic fuzzy supra topological space is considered an intuitionistic fuzzy soft supra topological space depend on only a parameter.

Proposition 3.3. *Let (X, τ, E) be an intuitionistic fuzzy soft supra topological space over X. Then the collection*

$$\tau_{1e} = \{F(e) : (F, G) \in \tau\}, \tau_{2e} = \left\{G'(e) : (F, G) \in \tau\right\}$$

defines two fuzzy supra topologies on X for each $e \in E$.

Note that τ_{1e} and τ_{2e} are independent of each other and $G'(e)$ is a complement of $G(e)$.

Example 3.3. Let $X = \{x_1, x_2\}$ and $E = \{e_1, e_2\}$. The intuitionistic fuzzy soft sets $(F_i, G_i) : E \to IFS(X)$ on X, for $1 \leq i \leq 3$, are defined as follows:

$(F_1, G_1)(e_1) = \left\langle \left(x_1, \left(\frac{1}{2}, \frac{1}{2}\right)\right), \left(x_2, \left(\frac{2}{3}, \frac{1}{3}\right)\right) \right\rangle,$
$(F_1, G_1)(e_2) = \left\langle \left(x_1, \left(\frac{1}{5}, \frac{4}{5}\right)\right), \left(x_2, \left(\frac{3}{5}, \frac{2}{5}\right)\right) \right\rangle,$
$(F_2, G_2)(e_1) = \left\langle \left(x_1, \left(\frac{1}{3}, \frac{2}{3}\right)\right), (x_2, (1, 0)) \right\rangle,$
$(F_2, G_2)(e_2) = \left\langle \left(x_1, \left(\frac{1}{7}, \frac{6}{7}\right)\right), \left(x_2, \left(\frac{4}{5}, \frac{1}{5}\right)\right) \right\rangle,$

$(F_3, G_3)(e_1) = \langle (x_1, (\frac{1}{2}, \frac{1}{2})), (x_2, (1, 0)) \rangle,$
$(F_3, G_3)(e_2) = \langle (x_1, (\frac{1}{5}, \frac{4}{5})), (x_2, (\frac{4}{5}, \frac{1}{5})) \rangle.$

Hence $\tau = \left\{ \widetilde{\Phi}, \widetilde{1}, (F_1, G_1), (F_2, G_2), (F_3, G_3) \right\}$ is an intuitionistic fuzzy soft supra topology on X.

$$\tau_{e_1} = \{(\underline{0}, \underline{1}), (\underline{1}, \underline{0}), (F_1, G_1)(e_1), \ (F_2, G_2)(e_1), (F_3, G_3)(e_1)\}$$

and

$$\tau_{e_2} = \{(\underline{0}, \underline{1}), (\underline{1}, \underline{0}), (F_1, G_1)(e_2), (F_2, G_2)(e_2), (F_3, G_3)(e_2)\}$$

are an intuitionistic fuzzy supra topologies on X for $e_1, e_2 \in E$. Also

$$\tau_{1e_1} = \{\underline{0}, \underline{1}, \ (F_1)(e_1), \ (F_2)(e_1), (F_3)(e_1)\},$$
$$\tau_{1e_2} = \{\underline{0}, \underline{1}, \ (F_1)(e_2), \ (F_2)(e_2), (F_3)(e_2)\},$$
$$\tau_{2e_1} = \left\{\underline{0}, \underline{1}, \ (G_1)'(e_1), \ (G_2)'(e_1), (G_3)'(e_1)\right\},$$
$$\tau_{2e_2} = \left\{\underline{0}, \underline{1}, \ (G_1)'(e_2), \ (G_2)'(e_2), (G_3)'(e_2)\right\}$$

are two fuzzy supra topologies on X.

Remark 3.4. The converse of the Proposition 3.3 is not true.

Proposition 3.4. *If τ is an intuitionistic fuzzy soft supra topology on X, then*

$$\gamma_\tau = \{T \in IFSS(X, E) : (\forall (F, G)) \ \ (F, G) \in \tau \Rightarrow T \cap (F, G) \in \tau\}$$

is an intuitionistic fuzzy soft topology on X and $\gamma_\tau \widetilde{\subset} \tau$.

Proof.

(1) $(F, G) \in \tau \Rightarrow \widetilde{\Phi} \cap (F, G) = \widetilde{\Phi}$ and $\widetilde{1} \cap (F, G) = (F, G) \in \tau$. Hence $\widetilde{\Phi}, \widetilde{1} \in \gamma_\tau$.

(2) Let $\{(F_i, G_i)\} \subset \gamma_\tau$ and $(F, G) \in \tau$. Since

$$\left(\widetilde{\cup}(F_i, G_i)\right) \cap (F, G) = (\vee F_i, \wedge G_i) \cap (F, G) = (\vee (F_i \wedge F), \wedge (G_i \vee G))$$
$$= \vee (F_i \wedge F, G_i \vee G) = \widetilde{\cup}((F_i, G_i) \cap (F, G)) \in \tau.$$

We have $\left(\widetilde{\cup}(F_i, G_i)\right) \in \gamma_\tau$.

(3) Let $(F_1, G_1), (F_2, G_2) \in \gamma_\tau$ and $(F, G) \in \tau$. Then
$((F_1, G_1)) \cap ((F_2, G_2)) \cap (F, G) = (F_1 \wedge F_2, G_1 \vee G_2) \cap (F, G)$
$= ((F_1 \wedge F_2) \wedge F, (G_1 \vee G_2) \vee G) = (F_1 \wedge (F_2 \wedge F), G_1 \vee (G_2 \vee G))$
$= (F_1, G_1) \cap ((F_2, G_2) \cap (F, G))$
Since $(F_2, G_2) \cap (F, G) \in \tau$
and $(F_1, G_1) \in \gamma_\tau, (F_1, G_1) \cap [(F_2, G_2) \cap (F, G)] \in \tau$.
Hence $(F_1, G_1) \cap (F_2, G_2) \in \gamma_\tau$.
Thus γ_τ is an intuitionistic fuzzy soft topology on X.
If $T \in \gamma_\tau$, then $T \in \tau$, since $\widetilde{1} \in \tau$ and $T \widetilde{\cap} \widetilde{1} = T \in \tau$.

□

Proposition 3.5. *Let (X, τ_1, E) and (X, τ_2, E) be two intuitionistic fuzzy soft supra topological spaces over the same universe X, then $\left(X, \tau_1 \widetilde{\cap} \tau_2, E\right)$ is an intuitionistic fuzzy soft supra topological space over X.*

Remark 3.5. The union of two intuitionistic fuzzy soft supra topologies on X may not be an intuitionistic fuzzy soft supra topology on X.

Definition 3.3. The intuitionistic fuzzy soft supra closure of $(F,G) \in IFSS$, denoted by $s-cl\,(F,G)$, is defined as intersection of all intuitionistic fuzzy soft supra closed sets containing (F,G).

Proposition 3.6. *Let (X,τ,E) be an intuitionistic fuzzy soft supra topological space on X, (F_1,G_1) and $(F_2,G_2) \in IFSS$. Then*

(1) $s-cl\left(\widetilde{\Phi}\right) = \widetilde{\Phi}$,
(2) $(F_1,G_1) \subset s-cl\,(F_1,G_1)$,
(3) (F_1,G_1) *is an intuitionistic fuzzy soft supra closed set if and only if* $(F_1,G_1) = s-cl\,(F_1,G_1)$,
(4) $s-cl\,(F_1,G_1) = s-cl\,(s-cl\,(F_1,G_1))$,
(5) $(F_1,G_1) \subset (F_2,G_2)$ *implies that* $s-cl\,(F_1,G_1) \subset s-cl\,(F_2,G_2)$,
(6) $s-cl\,(F_1,G_1) \cup s-cl\,(F_2,G_2) = s-cl\,((F_1,G_1) \cup (F_2,G_2))$.

Example 3.4. Let $X = \{x_1,x_2\}$ and $E = \{e_1,e_2\}$. The intuitionistic fuzzy soft sets $(F_i,G_i) : E \to IFS\,(X)$ on X, for $1 \leq i \leq 3$, are defined as follows:

$(F_1,G_1)(e_1) = \left\langle \left(x_1, \left(\frac{1}{5},\frac{1}{4}\right)\right), \left(x_2, \left(\frac{1}{2},\frac{1}{3}\right)\right) \right\rangle$,
$(F_1,G_1)(e_2) = \left\langle \left(x_1, \left(\frac{1}{4},\frac{2}{3}\right)\right), \left(x_2, (1,0)\right) \right\rangle$,
$(F_2,G_2)(e_1) = \left\langle \left(x_1, \left(\frac{1}{3},\frac{2}{3}\right)\right), \left(x_2, \left(\frac{1}{5},\frac{3}{5}\right)\right) \right\rangle$,
$(F_2,G_2)(e_2) = \left\langle \left(x_1, \left(\frac{1}{2},\frac{1}{4}\right)\right), \left(x_2, \left(\frac{1}{4},\frac{2}{3}\right)\right) \right\rangle$,
$(F_3,G_3)(e_1) = \left\langle \left(x_1, \left(\frac{1}{3},\frac{1}{4}\right)\right), \left(x_2, \left(\frac{1}{2},\frac{1}{3}\right)\right) \right\rangle$,
$(F_3,G_3)(e_2) = \left\langle \left(x_1, \left(\frac{1}{2},\frac{1}{4}\right)\right), \left(x_2, (1,0)\right) \right\rangle$.

$\tau = \left\{\widetilde{\Phi}, \widetilde{1}, (F_1,G_1),\ (F_2,G_2), (F_3,G_3)\right\}$ is an intuitionistic fuzzy soft supra topology on X. Let us consider

$(F,G)(e_1) = \left\langle \left(x_1, \left(\frac{1}{4},\frac{1}{3}\right)\right), \left(x_2, \left(\frac{3}{5},\frac{2}{5}\right)\right) \right\rangle$,
$(F,G)(e_2) = \left\langle \left(x_1, (0,1)\right), \left(x_2, \left(\frac{1}{3},\frac{1}{4}\right)\right) \right\rangle$.

Then $s-cl\,(F,G) = (F_2,G_2)'$ is obtained.

Definition 3.4. The intuitionistic fuzzy soft supra interior of $(F,G) \in IFSS$, denoted by $s-int\,(F,G)$, is defined union of all intuitionistic fuzzy soft supra open sets contained in (F,G).

Proposition 3.7. *Let (X,τ,E) be an intuitionistic fuzzy soft supra topological space on X, (F_1,G_1) and $(F_2,G_2) \in IFSS$. Then*

(1) $s-int\left(\widetilde{1}\right) = \widetilde{1}$,
(2) $s-int\,(F_1,G_1) \subset (F_1,G_1)$,
(3) (F_1,G_1) *is an intuitionistic fuzzy soft supra open set if and only if* $s-int\,(F_1,G_1) = (F_1,G_1)$,
(4) $s-int\,(F_1,G_1) = s-int\,(s-int\,(F_1,G_1))$,
(5) $(F_1,G_1) \subset (F_2,G_2)$ *implies that* $s-int\,(F_1,G_1) \subset s-int\,(F_2,G_2)$,
(6) $s-int\,(F_1,G_1) \cap s-int\,(F_2,G_2) \neq s-int\,((F_1,G_1) \cap (F_2,G_2))$.

Example 3.5. Let $X = \{x_1,x_2,x_3,x_4\}$ and $E = \{e_1\}$. The intuitionistic fuzzy soft sets $(F_i,G_i) : E \to IFS\,(X)$ on X, for $1 \leq i \leq 3$, are defined as follows:

$(F_1,G_1)(e_1)=\left\langle \left(x_1,\left(0,\frac{1}{2}\right)\right),\left(x_2,\left(\frac{1}{2},\frac{1}{2}\right)\right),\left(x_3,(1,0)\right),\left(x_4,\left(0,\frac{1}{2}\right)\right)\right\rangle,$
$(F_2,G_2)(e_1)=\left\langle \left(x_1,\left(\frac{1}{2},0\right)\right),\left(x_2,\left(\frac{1}{4},0\right)\right),\left(x_3,\left(0,\frac{1}{2}\right)\right),\left(x_4,(1,0)\right)\right\rangle,$
$(F_3,G_3)(e_1)=\left\langle \left(x_1,\left(\frac{1}{2},0\right)\right),\left(x_2,\left(\frac{1}{2},0\right)\right),\left(x_3,(1,0)\right),\left(x_4,(1,0)\right)\right\rangle.$

Then $\tau=\left\{\widetilde{\Phi},\widetilde{1},\ (F_1,G_1),\ (F_2,G_2),(F_3,G_3)\right\}$ is an intuitionistic fuzzy soft supra topology on X. Let
$\left(F^1,G^1\right)(e_1)=\left\langle \left(x_1,\left(\frac{1}{4},\frac{1}{2}\right)\right),\left(x_2,\left(\frac{1}{2},\frac{1}{2}\right)\right),\left(x_3,(1,0)\right),\left(x_4,\left(\frac{1}{2},\frac{1}{2}\right)\right)\right\rangle$ and
$\left(F^2,G^2\right)(e_1)=\left\langle \left(x_1,(1,0)\right),\left(x_2,\left(\frac{1}{2},0\right)\right),\left(x_3,\left(\frac{1}{2},0\right)\right),\left(x_4,(1,0)\right)\right\rangle$, then
$s-int\left(F^1,G^1\right)=(F_1,G_1)$ and $s-int\left(F^2,G^2\right)=(F_2,G_2)$, so
$s-int\left(F^1,G^1\right)\cap s-int\left(F^2,G^2\right)$
$=\left\langle \left(x_1,\left(0,\frac{1}{2}\right)\right),\left(x_2,\left(\frac{1}{4},\frac{1}{2}\right)\right),\left(x_3,\left(0,\frac{1}{2}\right)\right),\left(x_4,\left(0,\frac{1}{2}\right)\right)\right\rangle.$

But $\left(F^1,G^1\right)\cap\left(F^2,G^2\right)=\left\langle \left(x_1,\left(\frac{1}{4},\frac{1}{2}\right)\right),\left(x_2,\left(\frac{1}{2},\frac{1}{2}\right)\right),\left(x_3,\left(\frac{1}{2},0\right)\right),\left(x_4,\left(\frac{1}{2},\frac{1}{2}\right)\right)\right\rangle$ and so $s-int\left(\left(F^1,G^1\right)\cap\left(F^2,G^2\right)\right)=\widetilde{\Phi}$.

Definition 3.5. An intuitionistic fuzzy soft supra topological space (X,τ,E) is said to be a fuzzy strongly soft supra connected if X has no non-null intuitionistic fuzzy soft supra closed sets (F_1,G_1) and (F_2,G_2) in X such that $F_1(e)(x)+F_2(e)(x)\leq 1$ and $G_1(e)(x)+G_2(e)(x)\geq 1$, for each $e\in E$.

Remark 3.6. Let (X,τ,E) be a fuzzy strongly soft supra connected topological space. Then (X,τ_e) is an fuzzy strongly supra connected, for each $e\in E$.

Theorem 3.1. *Let (X,τ,E) be an $IFSSTS$. Then the following are equivalent:*

a) (X,τ,E) is a fuzzy strongly soft supra connected topological space,

b) There exists no intuitionistic fuzzy soft supra open sets (F_1,G_1) and (F_2,G_2) in X such that $(F_1,G_1)\neq\widetilde{1}\neq(F_2,G_2)$ and $F_1(e)(x)+F_2(e)(x)\geq 1$,for each $e\in E$.

c) There exists no $(F,G)\in\tau$ satisfying $(F,G)\neq\widetilde{1}$, $\frac{1}{2}\leq F(e)(x)$, and there exist no (F_1,G_1), $(F_2,G_2)\in\tau$ with $(F_1,G_1)\neq\widetilde{1}\neq(F_2,G_2)$ and $(F_1,G_1)\cup(F_2,G_2)=\widetilde{1}$.

Proof. a)⇒b) It is clear.

b)⇒c) First condition is obtained by considering $(F_1,G_1)=(F_2,G_2)=(F,G)$ and the other condition is a special case of b).

c)⇒a) If (X,τ,E) is not a fuzzy strongly soft supra connected topological space, then we have (F_1,G_1), $(F_2,G_2)\in\tau$ with $(F_1,G_1)\neq\widetilde{1}\neq(F_2,G_2)$ and $F_1(e)(x)+F_2(e)(x)\geq 1$,for each $e\in E$.

Then $(F,G)=(F_1,G_1)\cup(F_2,G_2)\in\tau,(F,G)\neq\widetilde{1}$ and $2F(e)(x)\geq F_1(e)(x)+F_2(e)(x)\geq 1$ which contradicts c), for each $e\in E$. □

Example 3.6. Let $X=\{x_1,x_2\}$ and $E=\{e_1\}$. The intuitionistic fuzzy soft sets on X are defined as follows

$(F_1,G_1)(e_1)=\left\langle \left(x_1,\left(\frac{1}{5},\frac{1}{4}\right)\right),\left(x_2,\left(\frac{1}{4},\frac{1}{2}\right)\right)\right\rangle,$
$(F_2,G_2)(e_1)=\left\langle \left(x_1,\left(\frac{1}{4},\frac{1}{2}\right)\right),\left(x_2,\left(\frac{1}{2},\frac{1}{5}\right)\right)\right\rangle,$
$(F_3,G_3)(e_1)=\left\langle \left(x_1,\left(\frac{1}{4},\frac{1}{4}\right)\right),\left(x_2,\left(\frac{1}{2},\frac{1}{5}\right)\right)\right\rangle.$

Then $\tau=\left\{\widetilde{\Phi},\widetilde{1},\ (F_1,G_1),\ (F_2,G_2),(F_3,G_3)\right\}$ is an intuitionistic fuzzy soft supra topology on X. There exist no intuitionistic fuzzy soft supra open sets $(F_i,G_i),(F_j,G_j)$ in X such that $F_i(e_1)(x)+F_j(e_1)(x_k)\geq 1$, $G_i(e_1)(x)+G_j(e_1)(x)\leq 1$, $i,j=1,2,3$ and $k=1,2$,for each $e\in E$.

Therefore (X,τ,E) is a fuzzy strongly soft supra connected topological space.

Definition 3.6. Let $(X,\tau,E),(Y,\tau',E)$ be two intuitionistic fuzzy soft supra topological spaces,$(F,G)\in IFSS$ on X, $(H,K)\in IFSS$ on Y and $f:(X,\tau,E)\to(Y,\tau',E)$ be a mapping. Then the image of (F,G) under the mapping f, denoted by

$$f((F,G))=(f(F),f(G))$$

is an intuitionistic fuzzy soft set over Y defined by

$$(f(F),f(G))(e)(y)=\left(\underset{f(x)=y}{\vee}F(e)(x),\underset{f(x)=y}{\wedge}G(e)(x)\right),$$

for each $e\in E$. Then the pre-image of (H,K) under the mapping f, denoted by

$$f^{-1}((H,K))=\left(f^{-1}(H),f^{-1}(K)\right)$$

is an intuitionistic fuzzy soft set over X defined by

$$\left(f^{-1}(H),f^{-1}(K)\right)(e)(x)=(H(e)(f(x)),K(e)(f(x))),$$

for each $e\in E$.

Definition 3.7. Let $(X,\tau,E),(Y,\tau',E)$ be two intuitionistic fuzzy soft supra topological spaces and $f:(X,\tau,E)\to(Y,\tau',E)$ be a mapping. Then the mapping f is said to be a fuzzy soft supra continuous if the pre-image of each $IFSS$ in τ' is in τ.

Theorem 3.2. *Let $(X,\tau,E),(Y,\tau',E)$ be two intuitionistic fuzzy soft supra topological spaces and $f:(X,\tau,E)\to(Y,\tau',E)$ be a mapping. Then the followings are equivalent:*

(1) f is a fuzzy soft supra continuous mapping,
(2) $f^{-1}((H,K))\in\tau$, $\forall(H,K)\in\tau'$,
(3) For each intuitionistic fuzzy soft supra closed set (H,K) on Y, $f^{-1}((H,K))$ is an intuitionistic fuzzy soft supra closed set on X,
(4) $f(s-cl(F,G))\subset s-cl(f(F,G)),\ \forall(F,G)\in(X,\tau,E)$,
(5) $s-cl\left(f^{-1}(F,G)\right)\subset f^{-1}(s-cl(F,G)),\ \forall(F,G)\in(Y,\tau',E)$,
(6) $f^{-1}(s-int(F,G))\subset s-int\left(f^{-1}(F,G)\right),\forall(F,G)\in(Y,\tau',E)$.

Theorem 3.3. *Let $(X,\tau,E),(Y,\tau',E)$ be two intuitionistic fuzzy soft supra topological spaces and $f:(X,\tau,E)\to(Y,\tau',E)$ be a surjective fuzzy soft supra continuous mapping. If (X,τ,E) is a fuzzy strongly soft supra connected topological space, then (Y,τ',E) is also a fuzzy strongly soft supra connected topological space.*

Proof. Suppose that (X,τ,E) is fuzzy strongly soft supra connected topological space but there exist $(F_1,G_1),(F_2,G_2)\in\tau'$ satisfying

$$(F_1,G_1)\neq\widetilde{1}_Y\neq(F_2,G_2)$$

and $F_1(e)(x)+F_2(e)(x)\geq 1$, for each $e\in E$.Then for all $x\in X$ we have

$$f^{-1}(F_1(e))(x)+f^{-1}(F_2(e))(x)=F_1(e)(f(x))+F_2(e)(f(x))\geq 1$$

which contradicts Theorem 3.1, since $f^{-1}((F_1,G_1)),f^{-1}((F_2,G_2))\in\tau$. It is clear that

$$f^{-1}(F_1,G_1)\neq\widetilde{1}_X\neq f^{-1}(F_2,G_2).$$

□

Definition 3.8. Let (X, τ, E) be an intuitionistic fuzzy soft supra topological space. If a family $\{(F_i, G_i) : i \in I\}$ of intuitionistic fuzzy soft supra open sets in X satisfies the condition

$$\tilde{\underset{i}{\cup}} (F_i, G_i) = \widetilde{1},$$

then it is called an intuitionistic fuzzy soft supra open cover of X.This means that

$$\underset{i}{\cup} (F_i, G_i)(e) = \underline{1}$$

is satisfied, for each $e \in E$. Here the family $\{(F_i, G_i)(e) : i \in I, e \in E\}$ is an intuitionistic fuzzy supra open cover. Then $\underset{i}{\vee} F_i(e) = \underline{1}$ and $\underset{i}{\wedge} G_i(e) = \underline{0}$ are holds, for each $e \in E$. So $\{F_i(e)\}$ and $\{G_i'(e)\}$ are two fuzzy open covers of X.

Definition 3.9. A family $\{(F_i, G_i) : i \in J \subset I\}$ is called an intuitionistic fuzzy soft supra open subcover of X. Here $\tilde{\underset{i \in J}{\cup}} (F_i, G_i) = \widetilde{1}$ is satisfied.

Definition 3.10. Let (X, τ, E) be an intuitionistic fuzzy soft supra topological space. (X, τ, E) is called an intuitionistic fuzzy soft supra compact topological space if every intuitionistic fuzzy soft supra open cover of X has a finite subcover.

Theorem 3.4. *Let (X, τ, E) and (Y, τ', E) be two intuitionistic fuzzy soft supra topological spaces and $f : (X, \tau, E) \to (Y, \tau', E)$ be a surjective fuzzy soft supra continuous mapping. If (X, τ, E) is intuitionistic fuzzy soft supra compact topological space, then so is (Y, τ', E).*

Definition 3.11. Let (X, τ, E) be an intuitionistic fuzzy soft supra topological space and (H, K) be an intuitionistic fuzzy soft set on X. Then

$$\tau_{(H,K)} = \{(H, K) \cap (F, G) : (F, G) \in \tau\}$$

is an intuitionistic fuzzy soft supra topology and $\left((H, K), \tau_{(H,K)}, E\right)$ is an intuitionistic fuzzy soft supra topological subspace.

Theorem 3.5. *Let (X, τ, E) be an intuitionistic fuzzy soft supra compact topological space and (F, G) is an intuitionistic fuzzy soft closed set on X. Then $\left((H, K), \tau_{(H,K)}, E\right)$ is an intuitionistic fuzzy soft supra compact topological space.*

4. Conclusion

We have introduced intuitionistic fuzzy soft supra topological spaces which are defined over an initial universe set with a fixed set of parameters. Then we investigated some of their important properties.

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Cigdem Gunduz Aras
Department of Mathematics, Kocaeli University, Kocaeli 41380, Turkey
E-mail address: caras@kocaeli.edu.tr

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