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DERIVATIVE FUNCTOR OF INVERSE LIMIT FUNCTOR IN THE CATEGORY OF NEUTROSOPHIC SOFT MODULES

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Abstract. This paper begins with the basic concepts of neutrosophic soft module. Later, we introduce inverse system in the category of neutrosophic soft modules and prove that its limit exists in this category. Generally, limit of inverse system of exact sequences of neutrosophic soft modules is not exact. Then we define the notion $\underleftarrow{\lim}^1$ which is first derived functor of the inverse limit functor. Finally, using methods of homology algebra, we prove that the inverse system limit of exact sequence of neutrosophic soft modules is exact.

1. Introduction

Many practical problems in economics, engineering, environment, social science, medical science etc. cannot be dealt with by classical methods, because classical methods have inherent difficulties. The reason for these difficulties may be due to the inadequacy of the theories of parameterizations tools.

The idea of extending the concept of fuzzy set to algebra dates back to the introduction in 1971 by Rosenfeld of fuzzy subgroups of a group [18]. Later several researchers have studied fuzzy modules and then Lopez-Permouth and Malik introduced the category of R-fz mod of fuzzy left $R-$modules over a ring R [12]. Ameri and Zahedi defined the concept of fuzzy exact sequence in the category of fuzzy modules, and obtained some results related to these notions [3, 23]. Same researchers have previously introduced the category of fuzzy (co) chain complexes and determined fuzzy homology functor in this category. It was proved that this functor is invariant with respect to fuzzy homotopy given in [3]. The intuitionistic fuzzy module has first been introduced by B. Davvaz [6].

Molodtsov [15] initiated the concept of soft set theory as a new mathematical tool for dealing with uncertainties. Later, work on the soft set theory is progressing rapidly. Maji et al. [14, 19] have published a detailed theoretical study on soft sets. After Molodtsov's work, some different applications of soft sets were studied in [1, 2, 8, 21].

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L. Jin-Liang [11] presented fuzzy soft sets and fuzzy soft groups. C. Gunduz and S. Bayramov [4, 5, 10] presented fuzzy soft modules and intuitionistic fuzzy soft modules.

The problems which obtained in new categories are closed according to algebraic operations is very important. Since inverse limit and direct limit contain most of the operations, the proof of presence of the limits is actual problem.

The inverse (direct) limit is not only an important concept in category theory, but also plays an important role in topology, algebra, homology theory etc. To the date, inverse and direct systems and their limits were defined in different categories. Furthermore, some of their properties were investigated [4, 9, 16].

This paper begins with the basic concepts of neutrosophic soft module. We introduce inverse system in the category of neutrosophic soft modules and prove that its limit exists in this category. Generally, limit of inverse system of exact sequences of neutrosophic soft modules is not exact. Then we define the notion $\varprojlim^1$ which is first derived functor of the inverse limit functor. Finally, using methods of homology algebra [5, 17] we prove that the inverse system limit of exact sequence of neutrosophic soft modules is exact.

2. Preliminaries

In this section, we recall necessary information commonly used in neutrosophic soft module.

Definition 2.1. [20] A neutrosophic set A on the universe of discourse X is defined as:

$$A = \{\langle x, T_{\mathrm{A}}(x), I_{\mathrm{A}}(x), F_{\mathrm{A}}(x)\rangle : x \in X\},$$

where $T,\ I,\ F : X \to]^{-}0,\ 1^{+}[$ *and* $^{-}0 \leq T_{\mathrm{A}}(x) + I_{\mathrm{A}}(x) + F_{\mathrm{A}}(x) \leq {}^{+}3$.

Definition 2.2. [15] Let X be an initial universe, E be a set of all parameters and $P(X)$ denotes the power set of X. A pair $(F,\ E)$ is called a soft set over X, where F is a mapping given by $F : E \to P(X)$.

Firstly, neutrosophic soft set defined by Maji [13] and later the concept has been modified by Deli and Broumi [7] as given below:

Definition 2.3. Let X be an initial universe set and E be a set of parameters. Let $P(X)$ denote the set of all neutrosophic sets of X. Then, a neutrosophic soft set $\left(\tilde{F}, E\right)$ over X is a set defined by a set valued function $\tilde{F}$ representing a mapping $\tilde{F} : E \to P(X)$ where $\tilde{F}$ is called approximate function of the neutrosophic soft set $\left(\tilde{F}, E\right)$. In other words, the neutrosophic soft set is a parameterized family of some elements of the set $P(X)$ and therefore it can be written as a set of ordered pairs,

$$\left(\tilde{F}, E\right) = \left\{ \left(e, \left\langle x, \mathrm{T}_{\tilde{F}(e)}(x), I_{\tilde{F}(e)}(x), F_{\tilde{F}(e)}(x), \right\rangle : x \in X\right) : e \in E\right\},$$

where $T_{\tilde{F}(e)}(x), I_{\tilde{F}(e)}(x), F_{\tilde{F}(e)}(x) \in [0,1]$, respectively, called the truth-membership, indeterminacy-membership, falsity- membership function of $\tilde{F}(e)$. Since supremum of each $T,\ I,\ F$ is 1 so the inequality $0 \leq \mathrm{T}_{\tilde{F}(e)}(x) + I_{\tilde{F}(e)}(x) + F_{\tilde{F}(e)}(x) \leq 3$ is obvious.

Definition 2.4. Let $\left(\tilde{F}_1, E\right)$ and $\left(\tilde{F}_2, E\right)$ be two neutrosophic soft sets over the common universe (X, E). Then their union is denoted by $\left(\tilde{F}_1, E\right) \bigcup \left(\tilde{F}_2, E\right) = = \left(\tilde{F}_3, E\right)$ and is defined by:

$$\left(\tilde{F}_3, E\right) = \left\{ \left(e, \left\langle x, T_{\tilde{F}_3(e)}(x), I_{\tilde{F}_3(e)}(x), F_{\tilde{F}_3(e)}(x) \right\rangle : x \in X\right) : e \in E \right\}$$

where

$$\begin{aligned} T_{\tilde{F}_3(e)}(x) &= \max\left\{T_{\tilde{F}_1(e)}(x), T_{\tilde{F}_2(e)}(x)\right\}, \\ I_{\tilde{F}_3(e)}(x) &= \max\left\{I_{\tilde{F}_1(e)}(x), I_{\tilde{F}_2(e)}(x)\right\}, \\ F_{\tilde{F}_3(e)}(x) &= \min\left\{F_{\tilde{F}_1(e)}(x) F_{\tilde{F}_2(e)}(x)\right\}. \end{aligned}$$

Definition 2.5. Let $\left(\tilde{F}_1, E\right)$ and $\left(\tilde{F}_2, E\right)$ be two neutrosophic soft sets over the common universe (X, E). Then their union is denoted by $\left(\tilde{F}_1, E\right) \bigcap \left(\tilde{F}_2, E\right) = = \left(\tilde{F}_3, E\right)$ and is defined by:

$$\left(\tilde{F}_3, E\right) = \left\{ \left(e, \left\langle x, T_{\tilde{F}_3(e)}(x), I_{\tilde{F}_3(e)}(x), F_{\tilde{F}_3(e)}(x) \right\rangle : x \in X\right) : e \in E \right\}$$

where

$$\begin{aligned} T_{\tilde{F}_3(e)}(x) &= \min\left\{T_{\tilde{F}_1(e)}(x), T_{\tilde{F}_2(e)}(x)\right\}, \\ I_{\tilde{F}_3(e)}(x) &= \min\left\{I_{\tilde{F}_1(e)}(x), I_{\tilde{F}_2(e)}(x)\right\}, \\ F_{\tilde{F}_3(e)}(x) &= \max\left\{F_{\tilde{F}_1(e)}(x), F_{\tilde{F}_2(e)}(x)\right\}. \end{aligned}$$

Definition 2.6. Let $\left\{ \left(\tilde{F}_i, E\right) | i \in I \right\}$ be a family of neutrosophic soft over the common universe (X, E). Then

$$\bigcup_{i\in I} \left(\tilde{F}_i, E\right) = \left\{\left(e, \left\langle x, \sup\left[T_{\tilde{F}_i(e)}(x)\right]_{i\in I}, \sup\left[I_{\tilde{F}_i(e)}(x)\right]_{i\in I}, \right.\right.\right.$$

$$\left.\left.\left. \inf\left[F_{\tilde{F}_i(e)}(x)\right]_{i\in I} \right\rangle : x \in X\right) : e \in E\right\},$$

$$\bigcap_{i\in I} \left(\tilde{F}_i, E\right) = \left\{\left(e, \left\langle x, \inf\left[T_{\tilde{F}_i(e)}(x)\right]_{i\in I}, \inf\left[I_{\tilde{F}_i(e)}(x)\right]_{i\in I}, \right.\right.\right.$$

$$\left.\left.\left. \sup\left[F_{\tilde{F}_i(e)}(x)\right]_{i\in I} \right\rangle : x \in X\right) : e \in E\right\},$$

Definition 2.7. Let $\left(\tilde{F}_1, E\right)$ and $\left(\tilde{F}_2, E\right)$ be two neutrosophic soft sets over the common universe (X, E). Then " AND" operation on them is denoted by $\left(\tilde{F}_1, E\right) \wedge \left(\tilde{F}_2, E\right) = \left(\tilde{F}_3, E \times E\right)$ and is defined by:

$$\left(\tilde{F}_3, E \times E\right) = \left\{ \left((e_1, e_2), \left\langle x, \mathrm{T}_{\tilde{F}_3(e_1,e_2)}(x), I_{\tilde{F}_3(e_1,e_2)}(x), F_{\tilde{F}_3(e_1,e_2)}(x), \right\rangle : \right.\right.$$

$$x \in X) : (e_1, e_2) \in E \times E\}$$

where

$$T_{\tilde{F}3(e_1,e_2)}(x) = \min\left\{T_{\tilde{F}1(e_1)}(x), T_{\tilde{F}2(e_2)}(x)\right\},$$
$$I_{\tilde{F}3(e_1,e_2)}(x) = \min\left\{I_{\tilde{F}1(e_1)}(x), I_{\tilde{F}2(e_2)}(x)\right\},$$
$$F_{\tilde{F}3(e_1,e_2)}(x) = \max\left\{F_{\tilde{F}1(e_{1,})}(x)\, F_{\tilde{F}2(e_2)}(x)\right\}.$$

Definition 2.8. Let $\left(\tilde{F}_1, E\right)$ and $\left(\tilde{F}_2, E\right)$ be two neutrosophic soft sets over the common universe (X, E). Then " OR" operation on them is denoted by $\left(\tilde{F}_1, E\right) \vee \left(\tilde{F}_2, E\right) = \left(\tilde{F}_3, E \times E\right)$ and is defined by:

$$\left(\tilde{F}_3, E \times E\right) = \left\{ \left((e_1, e_2), \left\langle x, T_{\tilde{F}3(e_1,e_2)}(x), I_{\tilde{F}3(e_1,e_2)}(x), F_{\tilde{F}3(e_1,e_2)}(x)\right\rangle \right.\right.$$
$$: x \in X) : (e_1, e_2) \in E \times E\}$$

where

$$T_{\tilde{F}3(e_1,e_2)}(x) = \max\left\{T_{\tilde{F}1(e_1)}(x), T_{\tilde{F}2(e_2)}(x)\right\},$$
$$I_{\tilde{F}3(e_1,e_2)}(x) = \max\left\{I_{\tilde{F}1(e_1)}(x), I_{\tilde{F}2(e_2)}(x)\right\},$$
$$F_{\tilde{F}3(e_1,e_2)}(x) = \min\left\{F_{\tilde{F}1(e_1)}(x)\, F_{\tilde{F}2(e_2)}(x)\right\}.$$

Definition 2.9. Let M be a left R-module and let $A = (T, I, F)$ be a neutrosophic set over M. Then we say (M, T, I, F) is a neutrosophic modul, if the following conditions are satisfied:

(1) $T(0) = I(0) = 1; \quad F(0) = 0$
(2) $T(x+y) \geq T(x) \wedge T(y);\ I(x+y) \geq I(x) \wedge I(y);$
$F(x+y) \leq \max\{F(x), F(y)\}$
(3) $T(\lambda x) \geq T(x);\ I(\lambda x) \geq I(x);\ F(\lambda x) \leq F(x)$

Definition 2.10. Let (M_1, T_1, I_1, F_1) and (M_2, T_2, I_2, F_2) be two neutrosophic modules over $M_1$and M_2, respectively. We say that f is a homomorphism of neutrosophic modules, if the following conditions for homomorphism of $f : M_1 \to M_2$ modules are satisfied:

$$T_2(f(x)) \geq T_1(x),\ I_2(f(x)) \geq I_1(x),\ F_2(f(x)) \leq F_1(x)$$

Let R be an ordinary ring, M be a left (or right) - module and $A \neq \emptyset$ be a set. $NS(M)$denotes the family of neutrosophic sets over M.

Definition 2.11. ([22]) Let $\left(\tilde{F}, A\right)$ be a neutrosophic soft set over M. Then $\left(\tilde{F}, A\right)$ is said to be a neutrosophic soft module over M if and only if $\forall a \in A$, $\tilde{F}(a) = (T_a, I_a, F_a)$ is a neutrosophic submodule of M and denoted as $\tilde{F}_a$.

Definition 2.12. ([22])Let $\left(\tilde{F}^1, A\right)$ and $\left(\tilde{F}^2, B\right)$ be two neutrosophic soft modules over M and N respectively, and let $f : M \to N$ be a homomorphism of modules, and let $g : A \to B$ be a mapping of sets. Then we say that $(f, g) : \left(\tilde{F}^1, A\right) \to \left(\tilde{F}^2, B\right)$ is a neutrosophic soft homomorphism of neutrosophic soft modules, if the following condition is satisfied:

$$f\left(T^1_{(a)}\right) = \tilde{F}^2(g(a)) = T^2_{g(a)},\ f\left(I^1_{(a)}\right) = \tilde{F}^2(g(a)) =$$

$$= I^2_{g(a)},\ f\left(F^1_{(a)}\right) = \tilde{F}^2\left(g\left(a\right)\right) = F^2_{g(a)}$$

Note that for $\forall a \in A$, $f : \left(M, \tilde{F}^1_{(a)}\right) \to \left(N, \tilde{F}^2_{g(a)}\right)$ is a neutrosophic homomorphism of neutrosophic modules.

Neutrosophic soft modules and their morphisms is consists of a category. This category is denoted by NSM.

Theorem 2.1. *([22]) Let* $\left(\tilde{F}^1, A\right)$ *and* $\left(\tilde{F}^2, A\right)$ *be two neutrosophic soft modules over* M*. Then* $\left(\tilde{F}^1, A\right) \wedge \left(\tilde{F}^2, B\right)$ *is a neutrosophic soft module over* M*.*

Theorem 2.2. *([22]) If* $\left\{\left(\tilde{F}_i, A_i\right)\right\}_{i\in I}$ *is a family of neutrosophic soft modules over* $\{M_i\}_{i\in I}$*, then* $\prod\limits_{i\in I}\left(\tilde{F}_i, A_i\right)$ *is a neutrosophic soft module over* $\prod\limits_{i\in I} M_i$*.*

Theorem 2.3. *([22]) If* $\left\{\left(\tilde{F}_i, A_i\right)\right\}_{i\in I}$ *is a family of neutrosophic soft modules over the family of modules* $\{M_i\}_{i\in I}$*, then* $\underset{i\in I}{\oplus}\left(\tilde{F}_i, A_i\right)$ *is a neutrosophic soft module over* $\underset{i\in I}{\oplus} M_i$*.*

3. Inverse system of neutrosophic soft modules

Definition 3.1. Any functor $D : \Lambda^{op} \to NSM$, where Λ is a directed set, is called an inverse system of neutrosophic soft modules.

Now we consider the following any inverse system

$$\left(\left\{(\tilde{F}_\alpha,\ A_\alpha)\right\}_{\alpha\in\Lambda},\ \left\{(p^{\alpha'}_\alpha, q^{\alpha'}_\alpha) : \left(\tilde{F'}_{\alpha'},\ A'_{\alpha'}\right) \to (\tilde{F}_\alpha, A_\alpha)\right\}_{\alpha\prec\alpha'}\right). \tag{3.1}$$

It is clear that parameter sets in (3.1) consist of the following inverse system of sets

$$\left(\{A_\alpha\}_{\alpha\in\Lambda},\ \left\{q^{\alpha'}_\alpha : A'_\alpha \to A_\alpha\right\}_{\alpha\prec\alpha'}\right). \tag{3.2}$$

Similarly, $\{M_\alpha\}_{\alpha\in\Lambda}$ in (3.1) consist of the following inverse system of modules

$$\left(\{M_\alpha\}_{\alpha\in\Lambda},\ \left\{p^{\alpha'}_\alpha : M'_\alpha \to M_\alpha\right\}_{\alpha\prec\alpha'}\right). \tag{3.3}$$

Let $A = \lim\limits_{\overleftarrow{\alpha}} A_\alpha$ be inverse limit of (3.2) and $M = \lim\limits_{\overleftarrow{\alpha}} M_\alpha$ be inverse limit of (3.3). Since $q^{\alpha'}_\alpha(a_{\alpha'}) = a_\alpha$ for all $a = \{a_\alpha\} \in A$,

$$\left(\left\{(M_\alpha,\ (\tilde{F}_\alpha)_{a_\alpha})\right\}_{\alpha\in\Lambda},\ \left\{p^{\alpha'}_\alpha : \left(M_{\alpha'},\ (\tilde{F}_{\alpha'})_{a_{\alpha'}}\right) \to (M_\alpha, (\tilde{F}_\alpha)_{a_\alpha})\right\}_{\alpha\prec\alpha'}\right) \tag{3.4}$$

is an inverse system of neutrosophic modules.

We denote inverse limit of (3.4) as $(M, \tilde{\Phi}_\alpha)$. We define $\tilde{\Phi} : A \to PN(M)$ as $\Phi(\alpha) = \tilde{\Phi}_\alpha$. Then $(\tilde{\Phi}, A)$ is a neutrosophic soft module over M.

If $\pi_\alpha : \lim\limits_{\leftarrow} M_\alpha \to M_\alpha$ and $q_\alpha : \lim\limits_{\leftarrow} A_\alpha \to A_\alpha$ are projection mappings, then $(\pi_\alpha, q_\alpha) : (\tilde{\Phi}, A) \to (\tilde{F}_\alpha, A_\alpha)$ is a homomorphism of neutrosophic soft modules, and, for $\alpha \prec \alpha'$, the following diagram is commutative:

$$\begin{array}{ccc} (\widetilde{\Phi}, A) & \xrightarrow{(\pi_\alpha, q_\alpha)} & (\widetilde{F}_\alpha, A_\alpha) \\ (\pi_{\alpha'}, q_{\alpha'}) \searrow & & \nearrow \left(P_\alpha^{\alpha'}, q_\alpha^{\alpha'}\right) \\ & (\widetilde{F}'_\alpha, A'_\alpha) & \end{array}$$

Theorem 3.1. *Every inverse system of neutrosophic soft modules has limit. This limit is unique and this limit is equal to* $(\widetilde{\Phi}, A)$.

Proof. We get inverse system (3.1). Let $(\tilde{G}, B)$ be a neutrosophic soft module over N. For $\left\{(h_\alpha, \varphi_\alpha) : (\tilde{G}, B) \to (\tilde{F}_\alpha, A_\alpha)\right\}_{\alpha\in\Lambda}$ be a family of neutrosophic soft homomorphism of neutrosophic soft modules, the conditions $\alpha \prec \alpha'$, $(p_\alpha^{\alpha'}, q_\alpha^{\alpha'})(h_{\alpha'}, \varphi_{\alpha'}) = (h_\alpha, \varphi_\alpha)$. Now we define neutrosophic soft homomorphism $(\psi, \gamma) : (\tilde{G}, B) \to (\widetilde{\Phi}, A)$, where $\gamma : B \to A = \lim\limits_{\overleftarrow{\alpha}} A_\alpha$, $\gamma(b) = \{\varphi_\alpha(b)\}$ and $\psi : N \to M = \lim\limits_{\overleftarrow{\alpha}} M_\alpha$, $\psi(x) = \{h_\alpha(x)\}$. Then $(\psi, \gamma) : (\tilde{G}, B) \to (\widetilde{\Phi}, A)$ is a neutrosophic soft homomorphism of neutrosophic soft modules. It is clear that for all $\alpha \in \Lambda$, the following diagram is commutative:

$$\begin{array}{ccc} (\widetilde{G}, B) & \xrightarrow{(h_\alpha, \varphi_\alpha)} & (\widetilde{F}_\alpha, A_\alpha) \\ (\psi, \gamma) \searrow & & \nearrow (\pi_\alpha, q_\alpha) \\ & (\widetilde{\Phi}, A) & \end{array}$$

□

The proof is completed.

Now we consider the following inverse system of neutrosophic soft modules over $\{N_\beta\}_{\beta\in\Lambda'}$

$$(\tilde{G}, B) = \left(\left\{(\tilde{G}_\beta, B_\beta)\right\}_{\beta\in\Lambda'}, \left\{\left(r_\beta^{\beta'},\ \chi_\beta^{\beta'}\right) : (\tilde{G}_{\beta'} B_{\beta'}) \to (\tilde{G}_\beta, B_\beta)\right\}_{\beta\prec\beta'}\right). \quad (3.5)$$

Let $\varphi : \Lambda' \to \Lambda$ be an isotone mapping and following mapping

$$(f_\beta, g_\beta) : \left(\tilde{F}_{\varphi(\beta)}, A_{\varphi(\beta)}\right) \to (\tilde{G}_\beta, B_\beta)$$

be a neutrosophic soft homomorphism of neutrosophic soft modules, for all $\beta \in \Lambda'$.

Definition 3.2. If for all $\beta \prec \beta'$, the condition

$$\left(r_\beta^{\beta'},\ \chi_\beta^{\beta'}\right) \circ (f'_\beta, g'_\beta) = (f_\beta, g_\beta) \circ \left(p_{\varphi(\beta)}^{\varphi(\beta')},\ q_{\varphi(\beta)}^{\varphi(\beta')}\right)$$

is satisfied, then the family $\left(\varphi,\ \{(f_\beta, g_\beta)\}_{\beta\in\Lambda'}\right)$ is said to by morphism of inverse systems.

It is clear that inverse systems of neutrosophic soft modules and morphisms of their consist of a category. This category is denoted as Inv(NSM).

Let $\left(\varphi,\ \{(f_\beta, g_\beta)\}_{\beta\in\Lambda'}\right) : (\underline{\tilde{F}}, \underline{A}) \to (\underline{\tilde{G}}, \underline{B})$ be a morphism of inverse systems of neutrosophic soft modules. Here $\underline{B} = \left(\{B_\beta\}_{\beta\in\Lambda'}, \left\{\chi_\beta^{\beta'}\right\}_{\beta\prec\beta'}\right)$ is an inverse system of sets and $\left(\varphi,\ \{(g_\beta)\}_{\beta\in\Lambda'}\right) : \underline{A} \to \underline{B}$ is a morphism of inverse systems of sets. Then the mapping $g = \varprojlim \left(\varphi, \{g_\beta\}_{\beta\in\Lambda'}\right) : \varprojlim_{\alpha} A_\alpha = A \to \varprojlim_{\beta} B_\beta = B$ is a mapping of limit sets of this inverse systems. Similarly,

$$\left(\varphi,\ \{(f_\beta)\}_{\beta\in\Lambda'}\right) : \{M_\alpha\}_{\alpha\in\Lambda} \to \{N_\beta\}_{\beta\in\Lambda'}$$

is a morphism of inverse systems of modules.

Proposition 3.1. Let $\varprojlim \left(\varphi, \{f_\beta\}_{\beta\in\Lambda'}\right) = f$. Then

$$(f, g) : \varprojlim_{\alpha}(\tilde{F}_\alpha, A_\alpha) \to \varprojlim_{\beta}(\tilde{G}_\beta, B_\beta)$$

is a morphism of limits of inverse systems of neutrosophic soft modules.

Proof. Since the product operation of neutrosophic soft modules is a functor, the following diagram is commutative:

$$\begin{array}{ccc} \prod\limits_{\beta} A_{\varphi(\beta)} & \underset{\longrightarrow}{\prod \widetilde{F}_\beta} & \prod\limits_{\beta} M_{\varphi(\beta)} \\ \prod g_\beta \downarrow & & \downarrow \prod f_\beta \\ \prod\limits_{\beta} B_\beta & \underset{\longrightarrow}{\prod \widetilde{G}_\beta} & \prod\limits_{\beta} N_\beta \end{array}$$

For all $\left\{\alpha_{\varphi(\beta)}\right\} \in \prod\limits_{\beta} A_{\varphi(\beta)}$

$$(\varphi, \{f_\beta\}_{\beta\in\Lambda'}) : \left\{\left(M_{\varphi(\beta)}, \tilde{F}_{a_{\varphi(\beta)}}\right)\right\} \to \left\{\left(N_\beta, \tilde{G}_{g\beta_{(a_{\varphi(\beta)})}}\right)\right\}_{\beta\in\Lambda'}$$

is a morphism of inverse systems of neutrosophic modules. Then

$$\varprojlim \left(\varphi, \{f_\beta\}_{\beta\in\Lambda'}\right) : \varprojlim \left\{\left(M_{\varphi(\beta)}, \tilde{F}_{a_{\varphi(\beta)}}\right)\right\} \to \varprojlim \left\{\left(N_\beta, \tilde{G}_{g\beta_{(a_{\varphi(\beta)})}}\right)\right\}_{\beta\in\Lambda'}$$

is a neutrosophic soft homomorphism of neutrosophic modules and the following diagram is commutative: □

$$\begin{array}{ccc} A & \overset{\widetilde{F}}{\rightarrow} & \varprojlim_{\beta} M_{\varphi(\beta)} \\ g \downarrow & & \downarrow f \\ B & \underset{\widetilde{G}}{\rightarrow} & \varprojlim_{\beta} N_\beta \end{array}$$

Theorem 3.2. *The corresponding*

$$\left\{\left(\tilde{F}_\alpha, A_\alpha\right)\right\}_{\alpha\in\Lambda} \to \varprojlim_{\alpha}\left(\tilde{F}_\alpha, A_\alpha\right)$$

is a covariant functor from the category Inv (NSM) to the category of NSM.

Theorem 3.3. *If* $\left\{(\tilde{F},A)\right\}_{j\in J}$ *is a family of inverse systems of neutrosophic soft modules, then*

$$\varprojlim \prod_j (\tilde{F},A)_j = \prod_j \varprojlim (\tilde{F},A)_j .$$

Proof. The proof of the theorem is straightforward. □

4. Derivative Functor of $\varprojlim$ functor

Let us review the problem of exact limit for inverse system of exact sequence of neutrosophic soft modules

Example 4.1. Let $M_n = \mathrm{Z}$, $M'_n = \mathrm{Z}$, $M''_n = \mathrm{Z}_2$ be modules over a ring. Then

$$\underline{M} = \left(\{M_n\}_{n\in N},\ \{p_n^{n+1}(m) = 3m\}\right)$$

$$\underline{M'} = \left(\left\{M'_n\right\}_{n\in N},\ \{q_n^{n+1}(m) = 3m\}\right)$$

$$\underline{M''} = \left(\left\{M''_n\right\}_{n\in N},\ \{r_n^{n+1}([m]) = [m]\}\right)$$

are inverse systems of modules and

$$f = \left\{f_n : M'_n \to M_n,\ f_n(m) = 2m\right\}$$

$$g = \left\{g_n : M_n \to M''_n,\ g_n(m) = [m]\right\}$$

are morphisms of inverse systems. The following sequence

$$0 \to \underline{M'} \xrightarrow{f} \underline{M} \xrightarrow{g} \underline{M''} \to 0$$

is short exact sequence of inverse systems of Z- modules.

Let A be a parameter set and

$$\tilde{F}'_n : A \to NSM(M'_n), \tilde{F}_n : A \to NSM(M_n), \tilde{F''}_n : A \to NSM(M''_n)$$

be three neutrosophic soft modules defined by the formula:

$$\forall a \in A, T'_{na} = (\chi(0))_{M'_n}, I'_{na} = (\chi(0))_{M'_n}, F'_{na} = 1 - (\chi(0))_{M'_n},$$

$$T_{na} = (\chi(0))_{M_n}, I_{na} = (\chi(0))_{M_n}, F_{na} = 1 - (\chi(0))_{M_n},$$

$$T''_{na} = (\chi(0))_{M''_n}, I''_{na} = (\chi(0))_{M''_n}, F''_{na} = 1 - (\chi(0))_{M''_n}.$$

The sequence

$$0 \to (M'_n, T'_{na}, I'_{na}, F'_{na}) \to (M_n, T_{na}, I_{na}, F_{na}) \to (M''_n, T''_{na}, I''_{na}, F''_{na}) \to 0$$

$$0 \to \left(M'_n, \tilde{F'}_n(a)\right) \xrightarrow{\overline{f_n}} \left(M_n, \tilde{F}_n(a)\right) \xrightarrow{\overline{g_n}} \left(M''_n, \tilde{F''}_n(a)\right) \to 0$$

is also short exact sequence of neutrosophic modules for each $a \in A$. Then the sequence

$$0 \to (\tilde{F'}, A) \to (\tilde{F}, A) \to (\tilde{F''}, A) \to 0$$

is short exact sequence of inverse systems of neutrosophic soft modules. Taking the limits of this sequence is not exact.

As it seen, the limit of inverse system of exact sequence of neutrosophic soft modules is not exact. So it is necessary to define derivative functor of inverse limit functor in category of neutrosophic soft modules.

We get inverse system in (3.1). We define the following homomorphism of modules

$$d : \prod_{\alpha} M_\alpha \to \prod_{\alpha} M_\alpha$$

by the formula:

$$d\left(\{x_\alpha\}\right) = \left\{x_\alpha - p_\alpha^{\alpha'}(x'_\alpha)\right\}_{\alpha \prec \alpha'}.$$

We demonstrate that $\forall a \in A$, d is a homomorphism of neutrosophic modules. Indeed,

$$T_{Aa}\left(d\left(\{x_\alpha\}\right)\right) = T_{Aa}\left(\left\{x_\alpha - p_\alpha^{\alpha'}(x_{\alpha'})\right\}\right) = \underset{\alpha}{\wedge} T_{\alpha\, a}\left(x_\alpha - p_\alpha^{\alpha'}(x_{\alpha'})\right)$$
$$\geq \underset{\alpha}{\wedge} \min\left\{\mathrm{T}_{\alpha\, a}(x_\alpha),\, T_{\alpha\, a}\left(p_\alpha^{\alpha'}(x_{\alpha'})\right)\right\}$$
$$I_{Aa}\left(d\left(\{x_\alpha\}\right)\right) = I_{Aa}\left(\left\{x_\alpha - p_\alpha^{\alpha'}(x_{\alpha'})\right\}\right) = \underset{\alpha}{\wedge} I_{\alpha\, a}\left(x_\alpha - p_\alpha^{\alpha'}(x_{\alpha'})\right)$$
$$\geq \underset{\alpha}{\wedge} \min\left\{\mathrm{I}_{\alpha\, a}(x_\alpha),\, I_{\alpha\, a}\left(p_\alpha^{\alpha'}(x_{\alpha'})\right)\right\};$$
$$F_{Aa}\left(d\left(\{x_\alpha\}\right)\right) = F_{Aa}\left(\left\{x_\alpha - p_\alpha^{\alpha'}(x_{\alpha'})\right\}\right) = \underset{\alpha}{\vee} F_{\alpha\, a}\left(x_\alpha - p_\alpha^{\alpha'}(x_{\alpha'})\right)$$
$$\leq \underset{\alpha}{\vee} \max\left\{F_{\alpha\, a}(x_\alpha),\, F_{\alpha\, a}\left(p_\alpha^{\alpha'}(x_{\alpha'})\right)\right\}.$$

Since $T_{\alpha\, a}\left(p_\alpha^{\alpha'}(x_{\alpha'})\right) \geq T_{\alpha' a}(x_{\alpha'})$, $I_{\alpha\, a}\left(p_\alpha^{\alpha'}(x_{\alpha'})\right) \geq I_{\alpha' a}(x_{\alpha'})$, and $F_{\alpha\, a}\left(p_\alpha^{\alpha'}(x_{\alpha'})\right) \leq F_{\alpha' a}(x_{\alpha'})$

$$T_{Aa}\left(d\left(\{x_\alpha\}\right)\right) \geq \underset{\alpha}{\wedge} \min\left\{T_{\alpha a}(x_\alpha),\ T_{\alpha' a}(x_{\alpha'})\right\} =$$
$$\underset{\alpha}{\wedge}\left(T_{\alpha a}(x_\alpha) \wedge\ T_{\alpha'' a}\left(x_{\alpha'}\right)\right) = \underset{\alpha}{\wedge} T_{\alpha a}(x_\alpha) = T_{Aa}\left(\{x_\alpha\}\right)$$
$$I_{Aa}\left(d\left(\{x_\alpha\}\right)\right) \geq \underset{\alpha}{\wedge} \min\left\{I_{\alpha a}(x_\alpha),\ I_{\alpha' a}(x_{\alpha'})\right\} =$$
$$\underset{\alpha}{\wedge}\left(I_{\alpha a}(x_\alpha) \wedge\ I_{\alpha'' a}\left(x_{\alpha'}\right)\right) = \underset{\alpha}{\wedge} I_{\alpha a}(x_\alpha) = I_{Aa}\left(\{x_\alpha\}\right)$$
$$F_{Aa}\left(d\left(\{x_\alpha\}\right)\right) \leq \underset{\alpha}{\vee} \max\left\{F_{\alpha a}(x_\alpha),\ F_{\alpha' a}(x_{\alpha'})\right\} =$$
$$\underset{\alpha}{\vee}\left(F_{\alpha a}(x_\alpha) \vee\ F_{\alpha'' a}\left(x_{\alpha'}\right)\right) = \underset{\alpha}{\vee} F_{\alpha a}(x_\alpha) = F_{Aa}\left(\{x_\alpha\}\right).$$

Then d is a homomorphism of neutrosophic modules. Therefore $\left(\ker d,\ \tilde{F}_{Aa}\,|\ker d\right)$ and $\left(co\ \ker d,\ \left(\tilde{F}_{Aa}\right)_p\right)$ are defined.

For inverse system of modules $\left(\{M_\alpha\}_{\alpha\in\Lambda},\ \left\{p_\alpha^{\alpha'}\right\}_{\alpha\prec\alpha'}\right)$, $\underset{\longleftarrow}{\lim}^{(1)} M_\alpha = \prod_\alpha M_\alpha \Big/ Imd$ is derivative functor.

If $\pi = \prod_\alpha M_\alpha \to \underset{\longleftarrow}{\lim}^{(1)} M_\alpha$ is the canonical homomorphism, we can define neutrosophic modules by $\left(\underset{\longleftarrow}{\lim}^{(1)} M_\alpha, (T_A)_a^\pi,\ (I_A)_a^\pi, (F_A)_{\pi a}\right)$. Then $(T_A^\pi, I_A^\pi, F_{\pi A}) : A \to \prod_\alpha M_\alpha$ is neutrosophic soft module.

Definition 4.1. $((T_A)^\pi, (I_A)^\pi, (F_A)_\pi)$ is called "first derived functor" of the inverse system of neutrosophic soft modules given (3.1).

Proposition 4.1. $\underset{\longleftarrow}{\lim}^{(1)}$ is a functor.

Proof. For this reason, it suffices to show that for each the morphism

$$\underline{\overline{f}} = \left(\rho : B \to A,\ \left\{(\overline{f}_\beta, g_\beta) : \left(\tilde{F}_{\rho(\beta)}, A_{\rho(\beta)}\right) \to (\tilde{G}_\beta, B_\beta)\right\}_{\beta\in B}\right),$$

$\underleftarrow{\lim}^{(1)}\underline{\overline{f}} : ((T_A)^\pi,\ (I_A)^\pi, (F_A)_\pi,\ A) \to \left((T^o_B)^\pi,\ (I^o_B)^\pi, (F^0_B)_\pi,\ B\right)$ is the homomorphism of neutrosophic soft modules. Since

$$(\tilde{F}^A_\pi)\,(x+imd) = \inf_{z\in Imd} \tilde{F}^A(x+z) \geq \inf_{z\in Imd} \tilde{G}_B(f(x+z)) = \inf_{z\in Imd} \tilde{G}_B(f(x)+f(z))$$

$$= \inf_{y=f(z)} \tilde{G}^B(f(x)+y) \geq \inf_{y\in Imd}(f(x)+y) = (\tilde{G}^B)_\pi \underleftarrow{\lim}^{(1)}\underline{f}(x+Imd)$$

$\underleftarrow{\lim}^{(1)}$ is a functor. □

We investigate another properties of $\underleftarrow{\lim}^{(1)}$ functor, let us introduce the category of chain complexes of neutrosophic soft modules.

Let $\left\{(\tilde{F}_n, A)\right\}_{n\in Z}$ be neutrosophic soft modules over $\{M_n\}_{n\in Z}$ and let for $\forall n \in Z$,

$$(\partial_n, 1_A) : \left(\tilde{F}_n, A\right) \to \left(\tilde{F}_{n-1}, A\right)$$

be homomorphism of neutrosophic soft modules.

Definition 4.2. If for all $a \in A$ $\{(M_n, T_{na},\ I_{na},\ F_{na})\,, \partial_n : (M_n, T_{na},\ I_{na},\ F_{na}) \to$ $\to \left(M_{n-1}, T_{n-1a},\ I_{n-1a},\ F_{n-1a}\right)\}$ is chain complex of neutrosophic soft modules, then the following sequence is said to be a chain complex of neutrosophic soft modules

$$(\tilde{F}, A) = \left\{\left(\tilde{F}_n, A\right), (\partial_n, 1_A) : \left(\tilde{F}_n,\ A\right) \to \left(\tilde{F}_{n-1}, A\right)\right\}.$$

Let $(\tilde{F}, A) = \left\{\left(\tilde{F}_n, A\right), (\partial_n, 1_A)\right\}$ be a chain complex of neutrosophic soft modules. Then for each $a \in A$ we obtain the neutrosophic homology module

$$H_n(\tilde{F}\,(a)) = \ker \partial_n \backslash Im\partial_{n+1}$$

for the neutrosophic chain complex

$$\left\{\left(M_n, \tilde{F}_n(a)\right), \partial_n : \left(M_n,\ \tilde{F}_n(a)\right) \to \left(M_{n-1}, \tilde{F}_{n-1}(a)\right)\right\}.$$

Thus, for all $a \in A$ the neutrosophic module $H_n(\tilde{F}\,(a))$ is a quotient module in $\left\{(M_n, \tilde{F}_{na})\right\}$. If there exists one to one covered connection with every neutrosophic submodule of neutrosophic quotient module of $(M_n, \tilde{F}_{na})$ and we think neutrosophic submodule of the neutrosophic module $H_n(\tilde{F}\,(a))$ as a neutrosophic submodule of $(M_n, \tilde{F}_{na})$. Thus,

$$H_n(\tilde{F}, -) : A \to NSM(M_n)$$

is a neutrosophic soft module.

Definition 4.3. Neutrosophic soft module $(H_n(\tilde{F}, -),\ A)$ is said to be $n-$ dimensional neutrosophic soft homology module of chain complex of neutrosophic soft modules

$$(\tilde{F}, A) = \left\{(\tilde{F}_n, A),\quad (\partial_n,\ 1_A)\right\}.$$

Definition 4.4. Let $\left\{(\tilde{F}_n, A), \quad (\partial_n, 1_A)\right\}$ and $\left\{(\tilde{G}_n, A), \quad (\partial'_n, 1_B)\right\}$ be two chain complexes of neutrosophic soft modules over $\{M_n\}_{n\in Z}$ and $\{N_n\}_{n\in Z}$, respectively and let $\{f_n : M_n \to N_n\}$ be homomorphism of modules, $g : A \to B$ be a mapping of sets. If for all $a \in A$, $f_n : (M_n, \tilde{F}_n^a) \to (N_n, \tilde{G}_n^{g(a)})$ is a neutrosophic homomorphism of neutrosophic modules and the condition $\partial'_n \circ f_n = f'_{n-1} \circ \partial_n$ is satisfied, then

$$(f_n, g) : (\tilde{F}_n, A) \to \left(\tilde{G}_n, A\right)$$

is said to be morphism of chain complexes of neutrosophic soft modules.

Definition 4.5. Let $(\{\varphi_n\}, g), (\{\psi_n\}, g) : \left\{(\tilde{F}_n, A), \partial_n\right\} \to \left\{(\tilde{G}_n, B), \partial'_n\right\}$ be morphism of chain complexes of neutrosophic soft modules and let

$$D = (\left\{\tilde{D}_n\right\}, g) : \left\{(\tilde{F}_n, A), \partial_n\right\} \to \left\{(\tilde{G}_{n+1}, B), \partial'_{n+1}\right\}$$

be a family of homomorphisms of neutrosophic soft modules. If the condition $\varphi_n - \psi_n = \tilde{D}_{n-1}\partial_n + \partial'_{n+1}\tilde{D}_n$ is satisfied, then the family of homomorphism of neutrosophic soft modules $D = (\left\{\tilde{D}_n\right\}, g)$ is said to be chain homotopy morphism. Also $(\{\varphi_n\}, g)$, $(\{\psi_n\}, g)$ is said to be chain homotopy mappings and denoted by $(\{\varphi_n\}, g) \sim (\{\psi_n\}, g)$.

The following theorem can be easily proved.

Theorem 4.1. *The chain homotopy relation is an equivalence relation and homology (cohomology) modules are invariant with respect to this relation.*

Let

$$\left(\left\{(\tilde{F}_\alpha\ A\right\}_{\alpha\in\Lambda}, \left\{\left(p_\alpha^{\alpha'}, 1_A\right) : \left(\tilde{F}_{\alpha'}, A\right) \to \left(\tilde{F}_\alpha, A\right)\right\}_{\alpha\prec\alpha'}\right)$$

be an inverse system of neutrosophic soft modules.

Let us consider the following cochain complex of neutrosophic soft modules

$$\bar{0} \to \left(\Pi\tilde{F}_\alpha, A\right) \xrightarrow{\bar{d}} \left(\Pi\tilde{F}_\alpha, A\right) \to \bar{0}.$$

Cohomology modules of this complex are $\ker\bar{d}$ and $co\ker\bar{d}$.

Lemma 4.1. $\lim\limits_{\leftarrow}(\tilde{F}_\alpha, A) = \ker\bar{d}$ *and* $\lim\limits_{\leftarrow}^{(1)}(\tilde{F}_\alpha, A) = co\ker\bar{d}$.

Proof. The proof of lemma is trivial. □

We accept natural numbers set which is index set of inverse system.

Theorem 4.2. *Let the sequence*

$$\left(\tilde{F}_1, A\right) \overset{p_1^2}{\leftarrow} \left(\tilde{F}_2, A\right) \overset{p_2^2}{\leftarrow} ...$$

be inverse sequence of neutrosophic soft modules. For each infinite subsequence of this sequence, $\lim\limits_{\longleftarrow}^{(1)}$ *dose not change.*

Proof. Let $S = \{i,\, j,\, k,\, ...\}$ be infinite subsequence of natural numbers N. From Lemma 4.1, $\underleftarrow{\lim}^{(1)}$ is defined by the following homomorphism of neutrosophic soft modules as appropriate subsequence

$$\bar{d}' : \left(\prod_{s\in S} \tilde{F}_s,\, A \right) \to \left(\prod_{s\in S} \tilde{F}_s,\, A \right).$$

We may define

$$f_0,\, f_1 : \prod_{s\in S} M_s \to \prod_{n\in N} M_n$$

homomorphisms of modules with this formula:

$$f_0(x_i, x_j, x_k, ...) = \left(p_1^i(x_i),\, p_2^i(x_i), ...,\, p_{i-1}^i(x_i),\, x_i,\, p_{i+1}^j(x_j), ..., p_{j-1}^j(x_j), x_j, ... \right)$$

$$f_1(x_i, x_j, x_k, ...) = (0,\ 0, ..., x_i,\, 0, ...,\, x_j,\, 0,\, ...,\, x_k,\ 0, ...)\,.$$

Also, for each $a \in A$

$$\begin{aligned}
&\left(\bigwedge_{n\in N} T_{na} \right) \left(p_1^i(x_i),\, ...,\, p_{i-1}^i(x_i),\, x_i,\, p_{i+1}^j(x_j), ..., p_{j-1}^j(x_j), x_j, ... \right) = \\
&= T_{1a}(p_1^i(x_i)) \wedge ... \wedge T_{i-1a}(p_{i-1}^i(x_i)) \wedge T_{ia}(x_i) \wedge T_{i+1a}(p_{i+1}^j(x_j)) \wedge ... \\
&\wedge T_j(x_j) \wedge ... \geq [T_{ia}(x_i) \wedge ... \wedge T_{ia}(x_i) \wedge T_{ia}(x_i)] \wedge [T_{ja}(x_j) \wedge ... \\
&\wedge T_{ja}(x_j)] \wedge ... = T_{ia}(x_i) \wedge T_{ja}(x_j) \wedge ... = \bigwedge_{s\in S} T_{sa}(x_s)
\end{aligned}$$

$$\begin{aligned}
&\left(\bigwedge_{n\in N} I_{na} \right) \left(p_1^i(x_i),\, ...,\, p_{i-1}^i(x_i),\, x_i,\, p_{i+1}^j(x_j), ..., p_{j-1}^j(x_j), x_j, ... \right) = \\
&= I_{1a}(p_1^i(x_i)) \wedge ... \wedge I_{i-1a}(p_{i-1}^i(x_i)) \wedge I_{ia}(x_i) \wedge I_{i+1a}(p_{i+1}^j(x_j)) \wedge ... \\
&\wedge I_j(x_j) \wedge ... \geq [I_{ia}(x_i) \wedge ... \wedge I_{ia}(x_i) \wedge I_{ia}(x_i)] \wedge [I_{ja}(x_j) \wedge ... \wedge I_{ja}(x_j)] \wedge ... = \\
&= I_{ia}(x_i) \wedge I_{ja}(x_j) \wedge ... = \bigwedge_{s\in S} I_{sa}(x_s)
\end{aligned}$$

$$\begin{aligned}
&\bigvee_{n\in N} F_{na} \left(p_1^i(x_i),\, ...,\, p_{i-1}^i(x_i),\, x_i,\, p_{i+1}^j(x_j), ..., p_{j-1}^j(x_j), x_j, ... \right) = \\
&= F_{1a}(p_1^i(x_i)) \vee ... \vee F_{i-1a}(p_{i-1}^i(x_i)) \vee F_{ia}(x_i) \vee F_{i+1a}(p_{i+1}^j(x_j)) \vee ... \vee F_j(x_j) \\
&\vee ... \leq [F_{ia}(x_i) \vee ... \vee F_{ia}(x_i) \vee F_{ia}(x_i)] \vee [F_{ja}(x_j) \vee ... \vee F_{ja}(x_j)] \vee ... = \\
&= F_{ia}(x_i) \vee F_{ja}(x_j) \vee ... = \bigvee_{s\in S} F_{sa}(x_s)
\end{aligned}$$

and

$$\begin{aligned}
&\left(\bigwedge_{n\in N} T_{na} \right) (0,\ 0,\ ...,\, x_i,\, x_i,\, 0, ..., x_j, 0, ...) = \\
&= T_{1a}(0) \wedge ... \wedge T_{ia}(x_i) \wedge T_{i+1a}(0) \wedge ... \wedge T_{ja}(x_j) \wedge ... = \\
&= T_{ia}(x_i) \wedge T_{ja}(x_i) \wedge ... = \bigwedge_{s\in S} T_{sa}(x_s),
\end{aligned}$$

$$\begin{aligned}
&\left(\bigwedge_{n\in N} I_{na} \right) (0,\ 0,\ ...,\, x_i,\, x_i,\, 0, ..., x_j, 0, ...) = \\
&= I_{1a}(0) \wedge ... \wedge I_{ia}(x_i) \wedge I_{i+1a}(0) \wedge ... \wedge I_{ja}(x_j) \wedge ... = \\
&= I_{ia}(x_i) \wedge I_{ja}(x_i) \wedge ... = \bigwedge_{s\in S} I_{sa}(x_s),
\end{aligned}$$

$$\begin{aligned}
&\left(\bigvee_{n\in N} F_{na} \right) (0,\ 0,\ ...,\, x_i,\, x_i,\, 0, ..., x_j, 0, ...) = \\
&= F_{1a}(0) \vee ... \vee F_{ia}(x_i) \vee F_{i+1a}(0) \vee ... \vee F_{ja}(x_j) \vee ... = \\
&= F_{ia}(x_i) \vee F_{ja}(x_i) \vee ... = \bigvee_{s\in S} F_{sa}(x_s)
\end{aligned}$$

□

Then $\bar{f}_0,\ \bar{f}_1 : \left(\prod_{s\in S} \tilde{F}_s,\ A\right) \to \left(\prod_{n\in N} \tilde{F}_n,\ A\right)$ are homomorphisms of neutrosophic soft modules. It is clear that the following diagram is commutative:

$$\begin{array}{ccc} \left(\prod\limits_{s\in S} \widetilde{F}_s, A\right) & \longrightarrow & \left(\prod\limits_{n\in N} \widetilde{F}_n, A\right) \\ \overline{d}' \downarrow & & \downarrow \overline{d} \\ \left(\prod\limits_{s\in S} \widetilde{F}_s, A\right) & \longrightarrow & \left(\prod\limits_{n\in N} \widetilde{F}_n, A\right) \end{array}$$

i.e. $\left\{\bar{f}_0,\ \bar{f}_1\right\}$ are morphisms of cochain complexes. Now, let us define

$$g_0,\ g_1 : \prod_{n\in N} M_n \to \prod_{s\in S} M_s$$

homomorphisms with this formula:

$$g_0(x_1, x_2,\ x_3, ...) = (x_i, x_j, x_k, ...)$$

$$g_1(x_1, x_2,\ x_3, ...) = \begin{pmatrix} x_i + p_i^{i+1}(x_{i+1}) + ... + p_i^{j-1}(x_{j-1}),\ x_j \\ +p_j^{j+1}(x_{j+1}) + ... + p_j^{k-1}(x_{k-1}), ... \end{pmatrix}.$$

For

$$\left(\wedge_{s\in S} T_{sa}\right)\ (x_i, x_j,\ x_k, ...) = T_{ia}(x_i)\ \wedge T_{ja}(x_j) \wedge ... \geq \wedge_{n\in N} T_{na}(x_n)$$

and

$$\left(\wedge_{s\in S} T_{sa}\right)\ \left(x_i + p_i^{i+1}(x_{i+1}) + ... + p_i^{j-1}(x_{j-1}),\ x_j + ... + p_j^{k-1}(\,x_{k-1}), ...\right) =$$

$$= T_{ia}\left(x_i + p_i^{i+1}(x_{i+1}) + ... + p_i^{j-1}(x_{j-1})\right) \wedge T_{ja}\ \left(x_j + ... + p_j^{k-1}(\,x_{k-1})\right)\ \wedge ... \geq$$

$$\geq \min\left\{T_{ia}\ \ (x_i)\,,\ T_{ia}\ \left(p_i^{i+1}(x_{i+1})\right), ...,\ T_{ia}\ \ \left(p_j^{j-1}(\,x_{j-1})\right)\right\} \wedge \min\left\{T_{ja}\ \ (x_j)\,, ...,\right.$$

$$\left.T_{ja}\ \left(p_j^{k-1}(x_{k-1})\right)\right\} \wedge .. \geq \min\left\{T_{ia}\ \ (x_i)\,,\ T_{i+1a}\ (x_{i+1})\,, ..., T_{j-1a}\ \ (\,x_{j-1})\right\} \wedge ...$$

$$\min\left\{T_{ja}\ \ (x_j)\,, ...,\ T_{j+1a}\ (x_{j+1})\,, ..., T_{k-1a}(x_{k-1})\right\} \wedge ... =$$
$$= \wedge_{m\in S} T_{ma}(x_m) \geq \wedge_{n\in N} T_{na}(x_n),$$

$$\left(\wedge_{s\in S} I_{sa}\right)\ (x_i, x_j,\ x_k, ...) = I_{ia}(x_i)\ \wedge I_{ja}(x_j) \wedge ... \geq \wedge_{n\in N} I_{na}(x_n)$$

$$\left(\wedge_{s\in S} I_{sa}\right)\ \left(x_i + p_i^{i+1}(x_{i+1}) + ... + p_i^{j-1}(x_{j-1}),\ x_j + ... + p_j^{k-1}(\,x_{k-1}), ...\right) =$$

$$= I_{ia}\ \left(x_i + p_i^{i+1}(x_{i+1}) + ... + p_i^{j-1}(x_{j-1})\right) \wedge\ I_{ja}\ \left(\ x_j + ... + p_j^{k-1}(\,x_{k-1})\right)\ \wedge ... \geq$$

$$\geq \min\left\{I_{ia}\ \ (x_i)\,,\ I_{ia}\ \left(p_i^{i+1}(x_{i+1})\right), ...,\ I_{ia}\ \ \left(p_j^{j-1}(x_{j-1})\right)\right\} \wedge$$

$$\min\left\{I_{ja}\ \left(x_j\right),...,\ I_{ja}\ \left(p_j^{k-1}(x_{k-1})\right)\right\}\wedge..\geq$$
$$\geq\min\left\{I_{ia}\ \left(x_i\right),\ I_{i+1a}\ \left(x_{i+1}\right),...,\ I_{j-1a}\ \left(x_{j-1}\right)\right\}\wedge...$$
$$\min\left\{I_{ja}\ \left(x_j\right),...,\ I_{j+1a}\ \left(x_{j+1}\right),...,I_{k-1a}(x_{k-1})\right\}\wedge...=$$
$$=\underset{m\in S}{\wedge}I_{ma}(x_m)\geq\underset{n\in N}{\wedge}I_{na}(x_n),$$

$$\left(\underset{s\in S}{\vee}F_{sa}\right)\ (x_i,x_j,\ x_k,...)=F_{ia}(x_i)\ \vee F_{ja}(x_j)\vee...\leq\underset{n\in N}{\vee}F_{na}(x_n)$$

and

$$\left(\underset{s\in S}{\vee}F_{sa}\right)\ \left(x_i+p_i^{i+1}(x_{i+1})+...+p_i^{j-1}(x_{j-1}),\ x_j+...+p_j^{k-1}(\ x_{k-1}),...\right)=$$
$$=F_{ia}\ \left(x_i+p_i^{i+1}(x_{i+1})+...+p_i^{j-1}(x_{j-1})\right)\vee\ F_{ja}\ \left(\ x_j+...+p_j^{k-1}(\ x_{k-1})\right)\ \vee..\leq$$
$$\leq\max\left\{F_{ia}\ \left(x_i\right),\ F_{ia}\ \left(p_i^{i+1}(x_{i+1})\right),...,\ F_{ia}\ \left(p_j^{j-1}(\ x_{j-1})\right)\right\}\vee\max$$
$$\left\{F_{ja}\ \left(x_j\right),...,\ F_{ja}\left(p_j^{k-1}(x_{k-1})\right)\right\}\vee...\leq$$
$$\leq\max\left\{F_{ia}\ \left(x_i\right),\ F_{i+1a}\ \left(x_{i+1}\right),...,\ F_{j-1a}\ \left(\ x_{j-1}\right)\right\}\vee..\vee$$
$$\max\left\{F_{ja}\ \left(x_j\right),...,\ F_{j+1a}\ \left(x_{j+1}\right),...,F_{k-1a}(x_{k-1})\right\}\vee..=$$
$$=\underset{m\in S}{\vee}F_{ma}(x_m)\leq\underset{n\in N}{\vee}F_{na}(x_n)$$

thus, $\bar{g}_0,\ \bar{g}_1:\left(\underset{n\in N}{\Pi}\tilde{F}_n,\ A\right)\rightarrow\left(\underset{s\in S}{\Pi}\tilde{F}_s,\ A\right)$ are homomorphisms of neutrosophic soft modules and $\bar{d}'\circ\bar{g}_0=\bar{g}_1\circ\bar{d}$ are satisfied, i.e., $\{\bar{g}_0,\ \bar{g}_1\}$ are homomorphisms of cochain complexes. It is clear that

$$\bar{g}_0\circ\bar{f}_0=\bar{g}_1\circ\bar{f}_1=\bar{1}_{\left(\underset{s\in S}{\Pi}\tilde{F}_s,A\right)}.$$

Hence, we give

$$D:\underset{n\in N}{\Pi}M_n\rightarrow\underset{n\in N}{\Pi}M_n$$

homomorphism of modules with this formula:

$$D(x_1,x_2,\ x_3,...)=\left(x_i+p_1^2(x_2)+...+p_1^{i-1}(x_{i-1}),\ x_2+p_2^3(x_3)+...+p_2^{i-1}(x_{i-1}),....\right.$$
$$\left.x_{i-1},\ 0,\ \ x_{i+1}+p_{i+1}^{i+2}(x_{i+2})+...+p_{i+1}^{j-1}(x_{j-1}),\ x_{i+2}+...+p_{i+2}^{j-1}(x_{j-1}),\ 0,\ ...\right)\ .$$

For,

$$\left(\underset{n\in N}{\wedge}T_{na}\right)\ (x_1+p_1^2(x_2)+...+p_1^{i-1}(x_{i-1}),\ x_2+p_2^3(x_3)+...$$
$$...+p_2^{i-1}(x_{i-1}),...,x_{i-1},0,...)=T_{1a}(x_1+p_1^2(x_2)+...+p_1^{i-1}(x_{i-1}))\wedge$$
$$\wedge T_{2a}(x_2+p_2^3(x_3)+...+p_2^{i-1}(x_{i-1}))\ \ \wedge...$$
$$\wedge T_{i-1a}(x_{i-1})\ \wedge\ T_{ia}(0)\ \wedge T_{i+1a}\left(x_{i+1}+p_{i+1}^{i+2}(x_{i+2})+...+p_{i+1}^{j-1}(x_{j-1})\right)\ \wedge...$$
$$\geq\min\left\{T_{1a}(x_1),\ T_{1a}\left(p_1^2(x_2)\right),...,T_{1a}\left(p_1^{i-1}(x_{i-1})\right)\right\}\wedge$$
$$\min\left\{T_{2a}(x_2),\ T_{2a}\left(p_2^3(x_3)\right),...,T_{2a}\left(p_2^{i-1}(x_{i-1})\right)\right\}\wedge T_{i-1a}(x_{i-1})\wedge1\wedge$$
$$\min\left\{T_{i+1a}(x_{i+1}),\ T_{i+1a}\left(p_{i+1}^{i+2}(x_{i+2})\right),...,T_{i+1a}\left(p_{i+1}^{j-1}(x_{j-1})\right)\right\}\wedge...$$
$$\geq\min\left\{T_{1a}(x_1),\ T_{2a}\left(x_2\right),...,T_{i-1a}\left(x_{i-1}\right)\right\}\wedge$$

$$\min\{T_{2a}(x_2),\, T_{3a}(x_3), ..., T_{i-1a}(x_{i-1})\} \wedge T_{i-1a}(x_{i-1}) \wedge T_{i+1a}(x_{i+1}) \wedge ...$$

$$= \underset{k=1}{\overset{i-1}{\wedge}} T_{ka}(x_k) \wedge \underset{k=2}{\overset{i-1}{\wedge}} T_{ka}(x_k) \wedge ... = \underset{n\in N}{\wedge} T_{na}(x_n),$$

$$\left(\underset{n\in N}{\wedge} I_{na}\right)\left(x_1 + p_1^2(x_2) + ... + p_1^{i-1}(x_{i-1}),\, x_2 + p_2^3(x_3) + ... + p_2^{i-1}(x_{i-1}), ..., x_{i-1}, 0, ...\right)$$

$$= I_{1a}(x_1 + p_1^2(x_2) + ... + p_1^{i-1}(x_{i-1})) \wedge I_{2a}(x_2 + p_2^3(x_3) + ... + p_2^{i-1}(x_{i-1})) \wedge ...$$

$$\wedge I_{i-1a}(x_{i-1}) \wedge I_{ia}(0) \wedge I_{i+1a}\left(x_{i+1} + p_{i+1}^{i+2}(x_{i+2}) + ... + p_{i+1}^{j-1}(x_{j-1})\right) \wedge ...$$

$$\geq \min\left\{I_{1a}(x_1),\, I_{1a}\left(p_1^2(x_2)\right), ..., I_{1a}\left(p_1^{i-1}(x_{i-1})\right)\right\} \wedge$$

$$\min\left\{I_{2a}(x_2),\, I_{2a}\left(p_2^3(x_3)\right), ..., I_{2a}\left(p_2^{i-1}(x_{i-1})\right)\right\} \wedge I_{i-1a}(x_{i-1}) \wedge 1 \wedge$$

$$\min\left\{I_{i+1a}(x_{i+1}),\, I_{i+1a}\left(p_{i+1}^{i+2}(x_{i+2})\right), ..., I_{i+1a}\left(p_{i+1}^{j-1}(x_{j-1})\right)\right\} \wedge ...$$

$$\geq \min\{I_{1a}(x_1),\, I_{2a}(x_2), ..., I_{i-1a}(x_{i-1})\} \wedge$$

$$\min\{I_{2a}(x_2),\, I_{3a}(x_3), ..., I_{i-1a}(x_{i-1})\} \wedge I_{i-1a}(x_{i-1}) \wedge I_{i+1a}(x_{i+1}) \wedge ...$$

$$= \underset{k=1}{\overset{i-1}{\wedge}} I_{ka}(x_k) \wedge \underset{k=2}{\overset{i-1}{\wedge}} I_{ka}(x_k) \wedge ... = \underset{n\in N}{\wedge} I_{na}(x_n),$$

$$\left(\underset{n\in N}{\vee} F_{na}\right)\left(x_1 + p_1^2(x_2) + ... + p_1^{i-1}(x_{i-1}),\, x_2 + p_2^3(x_3) + ... + p_2^{i-1}(x_{i-1}), ..., x_{i-1}, 0, ...\right)$$

$$= F_{1a}(x_1 + p_1^2(x_2) + ... + p_1^{i-1}(x_{i-1})) \vee F_{2a}(x_2 + p_2^3(x_3) + ... + p_2^{i-1}(x_{i-1})) \vee ...$$

$$\vee F_{i-1a}(x_{i-1}) \vee F_{ia}(0) \vee F_{i+1a}\left(x_{i+1} + p_{i+1}^{i+2}(x_{i+2}) + ... + p_{i+1}^{j-1}(x_{j-1})\right) \vee ...$$

$$\leq \max\left\{F_{1a}(x_1),\, F_{1a}\left(p_1^2(x_2)\right), ..., F_{1a}\left(p_1^{i-1}(x_{i-1})\right)\right\} \vee$$

$$\max\left\{F_{2a}(x_2),\, F_{2a}\left(p_2^3(x_3)\right), ..., F_{2a}\left(p_2^{i-1}(x_{i-1})\right)\right\} \vee F_{i-1a}(x_{i-1}) \vee 0 \vee$$

$$\max\left\{F_{i+1a}(x_{i+1}),\, F_{i+1a}\left(p_{i+1}^{i+2}(x_{i+2})\right), ..., F_{i+1a}\left(p_{i+1}^{j-1}(x_{j-1})\right)\right\} \vee ...$$

$$\leq \max\{F_{1a}(x_1),\, F_{2a}(x_2), ..., F_{i-1a}(x_{i-1})\} \vee$$

$$\max\{F_{2a}(x_2),\, F_{3a}(x_3), ..., F_{i-1a}(x_{i-1})\} \vee F_{i+1a}(x_{i-1}) \vee F_{i-1a}(x_{i-1}) \vee$$

$$\vee F_{i+1a}(x_{i+1}) \vee ... = \underset{k=1}{\overset{i-1}{\vee}} F_{ka}(x_k) \vee \underset{k=2}{\overset{i-1}{\vee}} F_{ka}(x_k) \vee ... = \underset{n\in N}{\vee} F_{na}(x_n).$$

$\bar{D} : \left(\prod_{n\in N} \tilde{F}_n,\, A\right) \to \left(\prod_{n\in N} \tilde{F}_n,\, A\right)$ is a homomorphism of neutrosophic soft modules. By using simplicity of calculation, it is shown that $\bar{D}$ is a chain homotopy between $\bar{f}_0 \circ \bar{g}_0$ and $\bar{f}_1 \circ \bar{g}_1$ homomorphisms. Then the following cohomology modules of cochain complexes

$$0 \to \left(\prod_{n\in N} \tilde{F}_n,\, A\right) \xrightarrow{\bar{d}} \left(\prod_{n\in N} \tilde{F}_n,\, A\right) \to 0$$

$$0 \to \left(\prod_{s\in S} \tilde{F}_s,\, A\right) \xrightarrow{\bar{d}} \left(\prod_{s\in S} \tilde{F}_s,\, A\right) \to 0$$

are neutrosophic soft isomorphic. Since $\lim\limits_{\leftarrow}$ is first cohomology module, the theorem is proved. Since $\underleftarrow{\lim}(\tilde{F}_n, A) = \ker \bar{d}$ and $p_n^{n+1}(x_{n+1}) = x_n$ is satisfied for each $\{x_n\} \in \underleftarrow{\lim} M_n$,

$$T_{na}(x_n) = T_{na}\left(p_n^{n+1}(x_{n+1})\right) \geq T_{n+1a}(x_{n+1})$$

$$I_{na}(x_n) = I_{na}\left(p_n^{n+1}(x_{n+1})\right) \geq I_{n+1a}(x_{n+1})$$

$$F_{na}(x_n) = F_{na}\left(p_n^{n+1}(x_{n+1})\right) \leq F_{n+1a}(x_{n+1})$$

i.e., for each $\{x_n\} \in \ker \bar{d}$, $\left\{\tilde{F}_{na}(x_n)\right\}$ is decreasing sequence.

Theorem 4.3. *For all* $\{x''_n\} \in \ker \bar{d}$, *if* $\lim\limits_{n\to\infty} T''_{na}(x''_n) = 0$ $\lim\limits_{n\to\infty} I''_{na}(x''_n) = 0$ *or* $\lim\limits_{n\to\infty} \tilde{F''}_{na}(x''_1) = 1$ *and the following diagram is short exact sequence of inverse system of neutrosophic soft modules*

$$\begin{array}{ccccccccc}
 & & \vdots & & \vdots & & \vdots & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \to & (\tilde{F'}_2,\ A) & \to & (\tilde{F}_2,\ A) & \to & (\tilde{F''}_2,\ A) & \to & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \to & (\tilde{F'}_1,\ A) & \to & (\tilde{F}_1,\ A) & \to & (\tilde{F''}_1,\ A) & \to & 0
\end{array}$$

then the sequence

$$0 \to \varprojlim(\tilde{F'}_n,\ A) \to \varprojlim(\tilde{F}_n,\ A) \to \varprojlim(\tilde{F''}_n,\ A) \to$$

$$\varprojlim{}^{(1)}(\tilde{F'}_n, A) \to \varprojlim{}^{(1)}(\tilde{F}_n, A) \to \varprojlim{}^{(1)}(\tilde{F''}_n,\ A) \to 0$$

is exact

Proof. For inverse system of neutrosophic soft modules $\left\{(\tilde{F}_n,\ A)\right\}_{n\in N}$,

$$C = 0 \xrightarrow{\bar{0}} \left(\prod_{n\in N} \tilde{F}_n,\ A\right) \xrightarrow{\bar{d}} \left(\prod_{n\in N} \tilde{F}_n,\ A\right) \xrightarrow{\bar{0}} 0 \xrightarrow{\bar{0}} \ldots$$

is a cochain complexes of neutrosophic soft modules.

$$H^0(C) = \varprojlim(\tilde{F}_n,\ A), H^1(C) = \varprojlim{}^{(1)}(\tilde{F}_n,\ A), H^k(C) = 0, k \geq 2 \tag{4.1}$$

are neutrosophic soft cohomology modules of this complexes. Similarly, for the inverse system of neutrosophic soft modules $\left\{(\tilde{F'}_n,\ A)\right\}$ and $\left\{(\tilde{F''}_n,\ A)\right\}$, we can constitute the following neutrosophic cochain complex

$$C' = 0 \xrightarrow{\bar{0}} \left(\prod_{n\in N} \tilde{F'}_n,\ A\right) \xrightarrow{\bar{d'}} \left(\prod_{n\in N} \tilde{F'}_n,\ A\right) \xrightarrow{\bar{0}} 0 \xrightarrow{\bar{0}} \ldots$$

$$C'' = 0 \xrightarrow{\bar{0}} \left(\prod_{n\in N} \tilde{F''}_n,\ A\right) \xrightarrow{\bar{d''}} \left(\prod_{n\in N} \tilde{F''}_n,\ A\right) \xrightarrow{\bar{0}} 0 \xrightarrow{\bar{0}} \ldots.$$

It is clear that neutrosophic cohomology modules of this complexes is the form in (4.1). From the condition of this theorem, the following sequence

$$0 \to C' \to C \to C'' \to 0$$

is short exact sequence of cochain complexes of neutrosophic soft modules. But generally, the following sequence of cohomology modules of this sequence

$$0 \to H^0(C') \to H^0(C) \to H^0(C'') \xrightarrow{\bar{\partial}} H^1(C')$$

$$\to H^1(C) \to H^1(C'') \to H^2(C') \to \cdots$$

is not exact, because $\bar{\partial}$ is usually not homomorphism of neutrosophic soft modules. Since $H^0(C'') = \ker d''$ and $\lim\limits_{n\to\infty} \tilde{F''}_{na}(x''_n) = 0$, grade function $\tilde{F''}$ of neutrosophic soft module $\left(H^0(C''),\ \tilde{F''}\right)$ is equal to grade function $\bar{0}$. Thus $\bar{\partial}$ is homomorphism of neutrosophic soft modules. Therefore the sequence

$$0 \to H^0(C') \to H^0(C) \to H^0(C'') \xrightarrow{\bar{\partial}} H^1(C')$$

$$\to H^1(C) \to H^1(C'') \to H^2(C') \to \cdots$$

is exact. By using (4.1), we obtain the following exact sequence of neutrosophic modules

$$0 \to \varprojlim(\tilde{F'}_n, A) \to \varprojlim(\tilde{F}_n, A) \to \varprojlim(\tilde{F''}_n, A) \to$$

$$\to \varprojlim{}^{(1)}(\tilde{F'}_n, A) \to \varprojlim{}^{(1)}(\tilde{F}_n, A) \to \varprojlim{}^{(1)}(\tilde{F''}_n, A) \to 0$$

□

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