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INTUITIONISTIC FUZZY TOPOLOGY ON SOFT SETS

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Abstract. The purpose of the paper is to define intuitionistic fuzzy topology (co-topology) (τ, τ^*) and examine some of its important properties. To do this, we firstly provide some fundamental properties of soft sets. Later in the paper, we introduce the concepts of base and subbase in intuitionistic fuzzy topological space of soft sets.

1. Introduction

To solve complicated problems in engineering, social sciences, economics and environment etc., we cannot use directly classical methods. Classical set theory, which is based on the crisp, may not be fully suitable for handling problems of uncertainty. A number of theories have been proposed for dealing with uncertainties. The concept of a fuzzy set was introduced by L.A. Zadeh in [23]. Various generalizations of fuzzy sets have been done by many researchers. The idea of fuzzy topological spaces was introduced by C.L Chang in [4]. Following this, Y.Yue and F. Jinming extended Lowen functors to $I-$fuzzy topological spaces in [22], Y. Yue gave $LM-$fuzzy topological spaces in [21]. He studied the stratifications of $LM-$fuzzy topologies. It is known that as a generalization of fuzzy sets, intuitionistic fuzzy sets were introduced by K. Atanassov [2]. T.K. Mondal and S. K. Samanta initiated concept of intuitionistic gradation of openness on fuzzy subsets of a nonempty set X in [16]. C. Liang and C. Yan defined base and subbase on intuitionistic $I-$fuzzy topological spaces in [11]. They also gave the base and subbase on the product of intuitionistic $I-$fuzzy topological spaces. There are other theories such as rough sets (see [18]), vague sets (see [5]) etc., which have their inherent difficulties. In 1999, D. Molodtsov introduced the concept of soft set theory which is completely a new approach for modeling uncertainty (see [15]). Since soft set theory has a rich potential, researches on soft set theory and its applications in various fields are progressing rapidly (see [12], [13]). The applications of soft set theory in algebraic structures were employed by H.Aktas and N. Cagman in [1]. They introduced soft groups and compared soft sets to fuzzy and rough sets. C. Gunduz(Aras) and S. Bayramov [7] defined intuitionistic fuzzy soft modules and investigated some important properties.

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Topological structures of soft sets have been studied by some authors in recent years. M. Shabir and M. Naz initiated the study of soft topological spaces which are defined over an initial universe with a fixed set of parameters in [19]. They showed that a soft topological space gives a parameterized family of topological spaces. Undoubtedly, soft topological spaces are an important generalization of topological spaces. Many authors studied on this theory and wrote a lot of papers related to it (see, e.g., [6, 8, 9, 14, 17, 20, 24]).

In this paper, we give the definition of intuitionistic fuzzy topology (cotopology) (τ, τ^*), which is a mapping satisfying some definite conditions from $SS(X, E)$ to $[0, 1]$. We show that an intuitionistic fuzzy topological space gives a parameterized family of soft bitopologies on X. Then we introduce the concepts of base and subbase of intuitionistic fuzzy topological spaces on soft sets.

2. Preliminaries

In this section we recall some necessary definitions for soft sets. Throughout this paper, X and E denote an initial universe set and a set of all parameters, respectively. By A we will denote a subset of E, i.e $A \subset E$.

Definition 2.1. [15] A pair (F, A) is called a soft set over X, where F is a mapping given by $F : A \to P(X)$.

In other words, soft set is a parameterized family of subsets of the set X. For $e \in A$, $F(e)$ may be considered as the set of $e-$elements of the soft set (F, A), i.e.,

$$(F, A) = \{(e, F(e)) : e \in A \subseteq E,\ F : A \to P(X)\}.$$

Afterwards, $SS(X, E)$ denotes the family of all soft sets over X with a fixed set of parameters E.

Definition 2.2. [13] For two soft sets (F, A) and (G, B) over X, (F, A) is called a soft subset of (G, B) if

(1) $A \subset B$ and
(2) $\forall e \in A$, $F(e)$ and $G(e)$ are identical approximations.

This relationship is denoted by $(F, A)\widetilde{\subseteq}(G, B)$. Similarly (F, A) is called a soft superset of (G, B) if (G, B) is a soft subset of (F, A). This relationship is denoted by $(F, A)\widetilde{\supseteq}(G, B)$. Two soft sets (F, A) and (G, B) over X are called soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 2.3. [13] The intersection of soft sets (F, A) and (G, B) over X is the soft set (H, C), where $C = A \cap B$ and $\forall e \in C$, $H(e) = F(e) \cap G(e)$. The soft set is denoted by $(F, A)\widetilde{\cap}(G, B) = (H, C)$.

Definition 2.4. [13] The union of soft sets (F, A) and (G, B) over X is the soft set, where $C = A \cup B$ and $\forall e \in C$,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B, \\ G(e), & \text{if } e \in B - A, \\ F(e) \cup G(e) & \text{if } e \in A \cap B. \end{cases}$$

The soft set is denoted by $(F, A)\widetilde{\cup}(G, B) = (H, C)$.

Definition 2.5. [13] A soft set (F,E) over X is said to be a null soft set, denoted by Φ, if $F(e)=\varnothing$ for all $e\in E$.

Definition 2.6. [13] A soft set (F,E) over X is said to be an absolute soft set, denoted by $\widetilde{X}$, if $F(e)=X$ for all $e\in E$.

Definition 2.7. [19] The difference of soft sets (F,E) and (G,E) over X , denoted by $(H,E)=(F,E)\backslash(G,E)$, if $H(e)=F(e)\backslash G(e)$ for all $e\in E$.

Definition 2.8. [19] The complement of a soft set (F,E), denoted by $(F,E)^c$, is defined $(F,E)^c=(F^c,E)$, where $F^c:E\to P(X)$ is a mapping given by $F^c(e)=X\backslash F(e)$ for all $e\in E$ and F^c is called the soft complement function of F.

Definition 2.9. [24] Let I be an arbitrary index set and $\{(F_i,E):i\in I\}$ be a subfamily of $SS(X,E)$. Then

$$\begin{aligned}\left(\underset{i\in I}{\cup}(F_i,E)\right)^c &= \underset{i\in I}{\cap}(F_i,E)^c,\\ \left(\underset{i\in I}{\cap}(F_i,E)\right)^c &= \underset{i\in I}{\cup}(F_i,E)^c.\end{aligned}$$

Definition 2.10. [10] Let (X,E) and $(Y,E^{'})$ be two soft sets, $f:X\to Y$ and $g:E\to E^{'}$ be two mappings and $(F,A)\subset(X,E)$. Then $(f_g):(X,E)\to(Y,E^{'})$ is called a soft mapping which is defined as: $(f_g)\left((F,A)\right)=f(F)_{g(a)}$ is a soft set in $(Y,E^{'})$ given by

$$f(F)(e^{'})=\begin{cases} f\left(\underset{e\in g^{-1}(e')\cap A}{\cup}F(e)\right), & \text{if } g^{-1}(e')\cap A\neq\emptyset,\\ \emptyset, & \text{otherwise,}\end{cases}$$

for $e^{'}\in B\subseteq E^{'}$where $B=g(A)\subseteq E^{'}$. $(f(F),g(A))$ is called soft image of (F,A).

Definition 2.11. [10] Let (X,E) and $(Y,E^{'})$ be two soft sets, $(f_g):(X,E)\to(Y,E^{'})$ be a soft mapping and $(G,C)\widetilde{\subseteq}(Y,E^{'})$. Then $(f_g)^{-1}\left((G,C)\right)=f^{-1}(G)_{g^{-1}(C)}$ is a soft set in (X,E) which is defined as:

$$f^{-1}(G)(e)=\begin{cases} f^{-1}\left(G(g(e))\right), & \text{if } g(e)\in C,\\ \emptyset, & \text{otherwise,}\end{cases}$$

for $e\in D\subseteq E$ where $D=g^{-1}(C)$. $(f_g)^{-1}\left((G,C)\right)$ is called soft inverse image of (G,C).

Definition 2.12. [19] Let τ be the collection of soft set over X. Then τ is said to be a soft topology on X if

1) $\Phi,\widetilde{X}$ belong to τ;
2) the union of any number of soft sets in τ belongs to τ;
3) the intersection of any two soft sets in τ belongs to τ.

The triplet (X,τ,E) is called a soft topological space over X.

Definition 2.13. [19] Let (X,τ,E) be a soft topological space over X. Then members of τ are said to be soft open sets in X.

Definition 2.14. [19] Let (X,τ,E) be a soft topological space over X. A soft set (F,E) over X is said to be a soft closed in X if its complement $(F,E)^c$ belongs to τ.

Definition 2.15. [3] Let (F,E) be a soft set over X. The soft set (F,E) is called a soft point, denoted by (x_e,E), if for the element $e\in E$, $F(e)=\{x\}$ and $F(e')=\varnothing$ for all $e'\in E-\{e\}$ (briefly denoted by x_e).

Definition 2.16. [9] A quadrable system (X,τ_1,τ_2,E) is called a soft bitopological space, where τ_1, τ_2 are arbitrary soft topologies on X and E be set of parameters.

3. Intuitionistic Fuzzy Topology on Soft Sets

Definition 3.1. A mapping $(\tau,\tau^*) : SS(X,E)\to[0,1]$ is called an intuitionistic fuzzy topology on X (briefly IFT) if the following conditions hold:

(i) $\tau(F,E)+\tau^*(F,E)\le 1$; $\forall (F,E)\in SS(X,E)$;
(ii) $\tau(\Phi)=\tau\left(\widetilde{X}\right)=1$, $\tau^*(\Phi)=\tau^*\left(\widetilde{X}\right)=0$;
(iii) $\tau\left((F,E)\,\widetilde{\cap}(G,E)\right)\ge\tau(F,E)\wedge\tau(G,E)$, $\tau^*\left((F,E)\,\widetilde{\cap}(G,E)\right)\le\tau^*(F,E)\vee\tau^*(G,E)$, $\forall (F,E),(G,E)\in SS(X,E)$;
(iv) $\tau\left(\underset{i\in\Delta}{\cup}(F_i,E)\right)\ge\underset{i\in\Delta}{\wedge}\tau(F_i,E)$, $\tau^*\left(\underset{i\in\Delta}{\cup}(F_i,E)\right)\le\underset{i\in\Delta}{\vee}\tau^*(F_i,E)$, $\forall (F_i,E)\in SS(X,E)$, $i\in\Delta$.

The quadruple (X,E,τ,τ^*) is called an intuitionistic fuzzy topological space of soft sets. Intuitionistic fuzzy topological space (X,E,τ,τ^*) is denoted by $IFTS$.

Example 3.1. Let $X=\mathbb{R}$, $E=\mathbb{N}$ and soft sets $F_k : E\to P(X)$ are defined as follows: for $\forall n\in\mathbb{N}$

$$\begin{aligned} F_1(n) &= [0,1],\\ F_2(n) &= [0,2],\\ &\ \ ...\\ F_k(n) &= [0,k],\\ &\ \ ... \end{aligned}$$

Now we consider $(\tau,\tau^*) : SS(X,E)\to[0,1]$ as follows:

$$\begin{aligned} \tau(F_k,E) &= 1-\frac{1}{k},\tau^*(F_k,E)=\frac{1}{k},\\ \tau(\Phi) &= \tau\left(\widetilde{X}\right)=1,\tau^*(\Phi)=\tau^*\left(\widetilde{X}\right)=0. \end{aligned}$$

From the definition of (τ,τ^*), (i) and (ii) are clear.
(iii) Let $k\le m$. Then

$$\begin{aligned} \tau\left((F_k,E)\,\widetilde{\cap}(F_m,E)\right) &= \tau(F_k,E)=1-\frac{1}{k},\\ \tau(F_k,E)\wedge\tau(F_m,E) &= \left(1-\frac{1}{k}\right)\wedge\left(1-\frac{1}{m}\right)=1-\frac{1}{k}. \end{aligned}$$

Thus $\tau\left((F_k,E)\,\widetilde{\cap}(F_m,E)\right)\geq\tau\left(F_k,E\right)\wedge\tau(F_m,E)$ is obtained.

$$\begin{aligned}\tau^*\left((F_k,E)\,\widetilde{\cap}(F_m,E)\right) &= \tau^*\left(F_k,E\right)=\frac{1}{k},\\ \tau^*\left(F_k,E\right)\vee\tau^*(F_m,E) &= \frac{1}{k}\vee\frac{1}{m}=\frac{1}{k},\end{aligned}$$

i.e. $\tau^*\left((F_k,E)\,\widetilde{\cap}(F_m,E)\right)\leq\tau^*\left(F_k,E\right)\vee\tau^*(F_m,E)$ is obtained.
(iv)

$$\begin{aligned}\tau\left(\underset{k\in\mathbb{N}}{\cup}(F_k,E)\right) &= \sup_{k\in\mathbb{N}}\left\{1-\frac{1}{k}\right\},\\ \underset{k\in\mathbb{N}}{\wedge}\tau(F_k,E) &= \inf_{k\in\mathbb{N}}\left\{1-\frac{1}{k}\right\}.\end{aligned}$$

So $\tau\left(\underset{k\in\mathbb{N}}{\cup}(F_k,E)\right)\geq\underset{k\in\mathbb{N}}{\wedge}\tau(F_k,E)$ is obtained.

$$\begin{aligned}\tau^*\left(\underset{k\in\mathbb{N}}{\cup}(F_k,E)\right) &= \inf_{k\in\mathbb{N}}\left\{\frac{1}{k}\right\},\\ \underset{k\in\mathbb{N}}{\vee}\tau^*(F_k,E) &= \sup_{k\in\mathbb{N}}\left\{\frac{1}{k}\right\}.\end{aligned}$$

Hence $\tau^*\left(\underset{k\in\mathbb{N}}{\cup}(F_k,E)\right)\leq\underset{k\in\mathbb{N}}{\vee}\tau^*(F_k,E)$. Then (τ,τ^*) is an intuitionistic fuzzy topology on X.

Definition 3.2. A mapping $(\upsilon,\upsilon^*):SS(X,E)\to[0,1]$ is called an intuitionistic fuzzy co-topology on X (briefly $IFCT$) if the following conditions hold:

(i$'$) $\upsilon\left(F,E\right)+\upsilon^*\left(F,E\right)\leq 1$; $\forall(F,E)\in SS(X,E)$;
(ii$'$) $\upsilon\left(\Phi\right)=\upsilon\left(\widetilde{X}\right)=1$, $\upsilon^*\left(\Phi\right)=\upsilon^*\left(\widetilde{X}\right)=0$;
(iii$'$) $\upsilon\left((F,E)\,\widetilde{\cup}(G,E)\right)\geq\upsilon\left(F,E\right)\wedge\upsilon\left(G,E\right)$, $\upsilon^*\left((F,E)\,\widetilde{\cup}(G,E)\right)\leq\upsilon^*\left(F,E\right)\vee\upsilon^*\left(G,E\right)$, $\forall(F,E),(G,E)\in SS(X,E)$;
(iv$'$) $\upsilon\left(\underset{i\in\Delta}{\cap}(F_i,E)\right)\geq\underset{i\in\Delta}{\wedge}\upsilon(F_i,E)$, $\upsilon^*\left(\underset{i\in\Delta}{\cap}(F_i,E)\right)\leq\underset{i\in\Delta}{\vee}\upsilon^*(F_i,E)$, $\forall(F_i,E)\in SS(X,E)$, $i\in\Delta$.

The quadruple $(X,E,\upsilon,\upsilon^*)$ is called an intuitionistic fuzzy co-topological space of soft sets. Intuitionistic fuzzy co-topological space (X,E,τ,τ^*) is denoted by $IFCTS$.

Theorem 3.1. *a) If (τ,τ^*) is an IFT on X, then (υ,υ^*) is an $IFCT$ on X such that $\upsilon\left(F,E\right)=\tau\left((F,E)^c\right),\upsilon^*\left(F,E\right)=\tau^*\left((F,E)^c\right)$.*

b) If (υ,υ^*) is an $IFCT$ on X, then (τ,τ^*) is an IFT on X such that $\tau\left(F,E\right)=\upsilon\left((F,E)^c\right),\tau^*\left(F,E\right)=\upsilon^*\left((F,E)^c\right)$.

Proof. a) Since $\upsilon\left(F,E\right)+\upsilon^*\left(F,E\right)=\tau\left((F,E)^c\right)+\tau^*\left((F,E)^c\right)\leq 1$, $\upsilon\left(F,E\right)+\upsilon^*\left(F,E\right)\leq 1$ is obtained, $\forall(F,E)\in SS(X,E)$. Clearly,

$$\upsilon\left(\Phi\right)=\tau\left(\Phi^c\right)=\tau\left(\widetilde{X}\right)=1,\ \upsilon\left(\widetilde{X}\right)=\tau\left(\widetilde{X}^c\right)=\tau\left(\Phi\right)=1,$$

$$\upsilon^*\left(\Phi\right)=\tau^*\left(\Phi^c\right)=\tau^*\left(\widetilde{X}\right)=0,\ \upsilon^*\left(\widetilde{X}\right)=\tau^*\left(\widetilde{X}^c\right)=\tau^*\left(\Phi\right)=0.$$

$\upsilon\left((F,E)\widetilde{\cup}(G,E)\right)=\tau\left(\left((F,E)\widetilde{\cup}(G,E)\right)^{c}\right)=\tau\left((F,E)^{c}\widetilde{\cap}(G,E)^{c}\right)$
$\geq\tau\left((F,E)^{c}\right)\wedge\tau\left((G,E)^{c}\right)=\upsilon\left(F,E\right)\wedge\upsilon\left(G,E\right).$

Similarly,
$\upsilon^{*}\left((F,E)\widetilde{\cup}(G,E)\right)=\tau^{*}\left(\left((F,E)\widetilde{\cup}(G,E)\right)^{c}\right)=\tau^{*}\left((F,E)^{c}\widetilde{\cap}(G,E)^{c}\right)$
$\leq\tau^{*}\left((F,E)^{c}\right)\vee\tau^{*}\left((G,E)^{c}\right)=\upsilon^{*}\left(F,E\right)\vee\upsilon^{*}\left(G,E\right),\ \forall(F,E),(G,E)\in SS(X,E).$

Now,
$\upsilon\left(\underset{i\in\Delta}{\cap}(F_i,E)\right)=\tau\left(\left(\underset{i\in\Delta}{\cap}(F_i,E)\right)^{c}\right)=\tau\left(\underset{i\in\Delta}{\cup}(F_i,E)^{c}\right)\geq\underset{i\in\Delta}{\wedge}\tau(F_i,E)^{c}=\underset{i\in\Delta}{\wedge}\upsilon(F_i,E),$

$\upsilon^{*}\left(\underset{i\in\Delta}{\cap}(F_i,E)\right)=\tau^{*}\left(\left(\underset{i\in\Delta}{\cap}(F_i,E)\right)^{c}\right)=\tau^{*}\left(\underset{i\in\Delta}{\cup}(F_i,E)^{c}\right)\leq\underset{i\in\Delta}{\vee}\tau^{*}(F_i,E)^{c}=\underset{i\in\Delta}{\vee}\upsilon^{*}(F_i,E).$

The proof is completed.

b) The proof is similar to a). □

Theorem 3.2. *Let (X,E,τ,τ^{*}) be an $IFTS$. Then for each $r\in(0,1]$,*

$$\begin{aligned}\tau_r &= \{(F,E)\in SS(X,E):\ \tau(F,E)\geq r\},\\ \tau_r^{*} &= \{(F,E)\in SS(X,E):\ \tau^{*}(F,E)\leq 1-r\}\end{aligned}$$

are descending families of soft topologies of soft sets on X such that $\tau_r\subset\tau_r^{*}$.

Proof. Since $\tau(\Phi)=\tau\left(\widetilde{X}\right)=1\geq r$, then $\Phi,\widetilde{X}\in\tau_r$. If $(F,E),(G,E)\in\tau_r$, $\tau\left((F,E)\widetilde{\cap}(G,E)\right)\geq\tau\left(F,E\right)\wedge\tau\left(G,E\right)\geq r$. Hence $(F,E)\widetilde{\cap}(G,E)\in\tau_r$. If $(F_i,E)\in\tau_r$, $\tau\left(\underset{i\in\Delta}{\cup}(F_i,E)\right)\geq\underset{i\in\Delta}{\wedge}\tau(F_i,E)\geq r$ for $i\in\Delta$. Then $\underset{i\in\Delta}{\cup}(F_i,E)\in\tau_r$. So τ_r is a soft topology for $\forall r\in(0,1]$. The proof of τ_r^{*} is similar to τ_r.

Suppose $(F,E)\in\tau_r$. Since $\tau\left(F,E\right)+\tau^{*}\left(F,E\right)\leq 1$, $\tau^{*}\left(F,E\right)\leq 1-\tau\left(F,E\right)\leq 1-r$. Hence $(F,E)\in\tau_r^{*}$. So $\tau_r\subset\tau_r^{*}$ is obtained. It is clear that $\{\tau_r\}_{r\in(0,1]}$ and $\{\tau_r^{*}\}_{r\in(0,1]}$ are descending families. □

Remark 3.1. Let (X,E,τ,τ^{*}) be an $IFTS$. Then intuitionistic fuzzy topological space gives a parameterized family of soft bitopologies on X for all $r\in(0,1]$.

Theorem 3.3. *Let $\{(\gamma_r,\gamma_r^{*})\}_{r\in(0,1]}$ be a descending family of soft bitopologies on X and $\gamma_r\subset\gamma_r^{*}$. Then*

$$\begin{aligned}\tau(F,E) &= \vee\{r:(F,E)\in\gamma_r\},\\ \tau^{*}(F,E) &= \wedge\{1-r:(F,E)\in\gamma_r^{*}\}\end{aligned}$$

are an $IFT's$.

Proof. Since $\Phi,\widetilde{X}\in\gamma_r,\gamma_r^{*}$, $\tau(\Phi)=\tau\left(\widetilde{X}\right)=1$ and $\tau^{*}(\Phi)=\tau^{*}\left(\widetilde{X}\right)=0$ are hold. Next let $(F,E),(G,E)\in SS(X,E)$, $\tau(F,E)=r_1$, $\tau(G,E)=r_2$ and $r=\min\{r_1,r_2\}$. If $r=0$, then $\tau\left((F,E)\widetilde{\cap}(G,E)\right)\geq 0=\tau\left(F,E\right)\wedge\tau\left(G,E\right)$. Suppose that $r>0$. Choose $\varepsilon>0$ such that $0<r-\varepsilon<r$. Then we choose $t_1,t_2\in(0,1)$ such that $r_1-\varepsilon<t_1$, $r_2-\varepsilon<t_2$ and $(F,E)\in\gamma_{t_1}$, $(G,E)\in\gamma_{t_2}$. Let $t=\min\{t_1,t_2\}$. Then $(F,E),(G,E)\in\gamma_t$ (since $\{\gamma_r\}_{r\in(0,1]}$ be a descending family). Hence $(F,E)\cap(G,E)\in\gamma_t$ (since γ_t is a soft topology). So, $\tau\left((F,E)\widetilde{\cap}(G,E)\right)\geq t\geq r-\varepsilon$. Since $\varepsilon>0$ is arbitrary, $\tau\left((F,E)\widetilde{\cap}(G,E)\right)\geq r=\tau\left(F,E\right)\wedge\tau\left(G,E\right)$.

Let $\{(F_i,E)\}_{i\in\Delta}$ be a family of soft sets, $P_i=\tau(F_i,E), i\in\Delta$ and $P=\underset{i\in\Delta}{\wedge}P_i$. If $P=0$, then

$$\tau\left(\underset{i\in\Delta}{\cup}(F_i,E)\right)\geq 0=\underset{i\in\Delta}{\wedge}\tau(F_i,E).$$

If $P>0$, choose $\varepsilon>0$ such that $P-\varepsilon>0$. For $i\in\Delta$, $\tau(F_i,E)\geq P>P-\varepsilon$. So there exists γ_r such that $(F_i,E)\in\gamma_r$ and $r\geq P-\varepsilon$. Since γ_r is a soft topology, $\underset{i\in\Delta}{\cup}(F_i,E)\in\gamma_r$. So

$$\tau\left(\underset{i\in\Delta}{\cup}(F_i,E)\right)\geq r>P-\varepsilon.$$

Since $\varepsilon>0$ is arbitrary,

$$\tau\left(\underset{i\in\Delta}{\cup}(F_i,E)\right)\geq P=\underset{i\in\Delta}{\wedge}\tau(F_i,E).$$

Now, let $(F,E),(G,E)\in SS(X,E)$, $\tau^*(F,E)=r_1$, $\tau^*(G,E)=r_2$ and $r=\max\{r_1,r_2\}$. If $r=1$, then $\tau^*\left((F,E)\widetilde{\cap}(G,E)\right)\leq 1=\tau^*(F,E)\vee\tau^*(G,E)$. Suppose $r<1$. Choose $\varepsilon>0$ such that $r+\varepsilon<1$. Then $\exists t_1,t_2\in(0,1)$ such that $t_1<r_1+\varepsilon$, $t_2<r_2+\varepsilon$ and $(F,E)\in\gamma^*_{1-t_1}$, $(G,E)\in\gamma^*_{1-t_2}$. Let $t=\max\{t_1,t_2\}$. Then $(F,E),(G,E)\in\gamma^*_{1-t}$ (since $\{\gamma^*_r\}$ is a descending family). So $(F,E)\widetilde{\cap}(G,E)\in\gamma^*_{1-t}$ (since γ^*_{1-t} is a soft topology). Hence

$$\tau^*\left((F,E)\widetilde{\cap}(G,E)\right)\leq t\leq r+\varepsilon.$$

Since $\varepsilon>0$ is arbitrary,

$$\tau^*\left((F,E)\widetilde{\cap}(G,E)\right)\leq r=\tau^*(F,E)\vee\tau^*(G,E).$$

Let $\{(F_i,E)\}_{i\in\Delta}$ be a family of soft sets, $P_i=\tau^*(F_i,E), i\in\Delta$ and $P=\underset{i\in\Delta}{\vee}P_i$.If $P=1$, then

$$\tau^*\left(\underset{i\in\Delta}{\cup}(F_i,E)\right)\leq 1=\underset{i\in\Delta}{\vee}\tau^*(F_i,E).$$

So, consider the case when $P<1$. Choose $\varepsilon>0$ such that $P+\varepsilon<1$. For $i\in\Delta$,

$$\tau^*\left(\underset{i\in\Delta}{\cup}(F_i,E)\right)\leq P<P+\varepsilon.$$

So we find γ^*_r such that $(F_i,E)\in\gamma^*_r$ and $1-r<P+\varepsilon$. Therefore

$$(F_i,E)\in\gamma^*_r\subset\gamma^*_{1-P-\varepsilon},\ \text{for}\ \forall i\in\Delta.$$

Since $\gamma^*_{1-P-\varepsilon}$ is a soft topology, $\underset{i\in\Delta}{\cup}(F_i,E)\in\gamma^*_{1-P-\varepsilon}$. Then

$$\tau^*\left(\underset{i\in\Delta}{\cup}(F_i,E)\right)\leq P+\varepsilon.$$

Since $\varepsilon>0$ is arbitrary,

$$\tau^*\left(\underset{i\in\Delta}{\cup}(F_i,E)\right)\leq P=\underset{i\in\Delta}{\vee}\tau^*(F_i,E).$$

Now we prove that $\tau(F,E)+\tau^*(F,E)\leq 1$ for $\forall(F,E)\in SS(X,E)$.

Let $\tau(F,E)=P$. If $P=0, \tau(F,E)+\tau^*(F,E)\leq 1$ is satisfied. If $P=1$, the soft set (F,E) belongs to $\gamma_r\subset\gamma^*_r$. Then $\tau^*(F,E)=0$ and $\tau(F,E)+\tau^*(F,E)\leq 1$. Next consider the case when $0<P<1$. Choose $\varepsilon>0$ such that $0<P-\varepsilon<$

$P < P + \varepsilon < 1$. Then $(F,E) \in \gamma_{P-\varepsilon} \subset \gamma^*_{P-\varepsilon}$ and $\tau^*(F,E) \leq 1 - P + \varepsilon$. So $\tau(F,E) + \tau^*(F,E) \leq 1 + \varepsilon$. Since $\varepsilon > 0$ is arbitrary, $\tau(F,E) + \tau^*(F,E) \leq 1$. (τ, τ^*) is an intuitionistic fuzzy topology on X. □

Definition 3.3. Let (X, E, τ, τ^*) be an $IFTS$.

a) $(\beta, \beta^*) : SS(X,E) \to [0,1]$ is called a base of (τ, τ^*) if the following conditions hold:

$$\forall (F,E) \in SS(X,E),\ \tau(F,E) = \bigvee_{\underset{i\in\Delta}{\cup}(G_i,E)=(F,E)} \bigwedge_{i\in\Delta} \beta(G_i,E)$$

and

$$\tau^*(F,E) = \bigwedge_{\underset{i\in\Delta}{\cup}(G_i,E)=(F,E)} \bigvee_{i\in\Delta} \beta^*(G_i,E).$$

b) $(\varphi, \varphi^*) : SS(X,E) \to [0,1]$ is called a subbase of (τ, τ^*) if $(\widetilde{\varphi}, \widetilde{\varphi}^*) : SS(X,E) \to [0,1]$ is a base of (τ, τ^*), where

$$\widetilde{\varphi}(F,E) = \bigvee_{\underset{j\in J}{\cap}(G_j,E)=(F,E)} \bigwedge_{j\in J} \varphi(G_j,E),$$

$$\widetilde{\varphi}^*(F,E) = \bigwedge_{\underset{j\in J}{\cap}(G_j,E)=(F,E)} \bigvee_{j\in J} \varphi^*(G_j,E)$$

and J is a finite set.

Theorem 3.4. *Define a map* $(\beta, \beta^*) : SS(X,E) \to [0,1]$ *as follows:*

a) $\beta(\Phi) = \beta\left(\widetilde{X}\right) = 1,\ \beta^*(\Phi) = \beta^*\left(\widetilde{X}\right) = 0$;

b) $\beta\left((F,E)\,\widetilde{\cap}(G,E)\right) \geq \beta(F,E) \wedge \beta(G,E)$, $\beta^*\left((F,E)\,\widetilde{\cup}(G,E)\right) \leq \beta^*(F,E) \vee \beta^*(G,E)$, $\forall (F,E),(G,E) \in SS(X,E)$.

Then

$$\tau_\beta(F,E) = \bigvee_{\underset{j\in J}{\cup}(G_j,E)=(F,E)} \bigwedge_{j\in J} \beta(G_j,E),$$

$$\tau^*_{\beta^*}(F,E) = \bigwedge_{\underset{j\in J}{\cup}(G_j,E)=(F,E)} \bigvee_{j\in J} \beta^*(G_j,E)$$

is an intuitionistic fuzzy topology and (β, β^*) is a base of $\left(\tau_\beta, \tau^*_{\beta^*}\right)$.

Proof. From the condition a) $\tau_\beta(\Phi) = \tau_\beta\left(\widetilde{X}\right) = 1, \tau^*_{\beta^*}(\Phi) = \tau^*_{\beta^*}\left(\widetilde{X}\right) = 0$ hold. For $\forall (F,E),(G,E) \in SS(X,E)$,

$$\begin{aligned}\tau_\beta(F,E) \wedge \tau_\beta(G,E) \;&=\; \left(\bigvee_{\underset{\alpha\in A}{\cup}(F_\alpha,E)=(F,E)} \bigwedge_{\alpha\in A} \beta(F_\alpha,E)\right) \\ &\quad \wedge \left(\bigvee_{\underset{\beta\in B}{\cup}(G_\beta,E)=(G,E)} \bigwedge_{\beta\in B} \beta(G_\beta,E)\right)\end{aligned}$$

$$
\begin{aligned}
&= \bigvee_{\underset{\alpha\in A}{\cup}(F_\alpha,E)=(F,E)} \bigvee_{\underset{\beta\in B}{\cup}(G_\beta,E)=(G,E)} \left(\left(\bigwedge_{\alpha\in A}\beta(F_\alpha,E)\right)\wedge\left(\bigwedge_{\beta\in B}\beta(G_\beta,E)\right)\right)\\
&\leq \bigvee_{\underset{\alpha\in A,\beta\in B}{\cup}\left((F_\alpha,E)\widetilde{\cap}(G_\beta,E)\right)=(F,E)\widetilde{\cap}(G,E)} \left(\bigwedge_{\alpha\in A,\beta\in B}\beta\left((F_\alpha,E)\widetilde{\cap}(G_\beta,E)\right)\right)\\
&\leq \bigvee_{\underset{\gamma\in C}{\cup}(H_\gamma,E)=(F,E)\widetilde{\cap}(G,E)} \bigwedge_{\gamma\in C}\beta(H_\gamma,E)\\
&= \tau_\beta\left((F,E)\,\widetilde{\cap}\,(G,E)\right)
\end{aligned}
$$

is obtained. Similarly,

$$
\begin{aligned}
\tau^*_{\beta^*}(F,E)\vee\tau^*_{\beta^*}(G,E) &= \left(\bigwedge_{\underset{\alpha\in A}{\cup}(F_\alpha,E)=(F,E)}\bigvee_{\alpha\in A}\beta^*(F_\alpha,E)\right)\\
&\quad\vee\left(\bigwedge_{\underset{\beta\in B}{\cup}(G_\beta,E)=(G,E)}\bigvee_{\beta\in B}\beta^*(G_\beta,E)\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \bigwedge_{\underset{\alpha\in A}{\cup}(F_\alpha,E)=(F,E)} \bigwedge_{\underset{\beta\in B}{\cup}(G_\beta,E)=(G,E)} \left(\left(\bigvee_{\alpha\in A}\beta^*(F_\alpha,E)\right)\vee\left(\bigvee_{\beta\in B}\beta^*(G_\beta,E)\right)\right)\\
&\geq \bigwedge_{\underset{\alpha\in A,\beta\in B}{\cup}\left((F_\alpha,E)\widetilde{\cap}(G_\beta,E)\right)=(F,E)\widetilde{\cap}(G,E)} \left(\bigvee_{\alpha\in A,\beta\in B}\beta^*\left((F_\alpha,E)\widetilde{\cap}(G_\beta,E)\right)\right)\\
&\geq \bigwedge_{\underset{\gamma\in C}{\cup}(H_\gamma,E)=(F,E)\widetilde{\cap}(G,E)} \bigvee_{\gamma\in C}\beta^*(H_\gamma,E)\\
&= \tau^*_{\beta^*}\left((F,E)\,\widetilde{\cap}\,(G,E)\right)
\end{aligned}
$$

is satisfied. Now, let $\{(F_\lambda,E):\lambda\in K\}$ be a family of soft sets. We consider a family

$$B_\lambda=\left\{\{(G_{\delta_\lambda},E):\delta_\lambda\in K_\lambda\}:\bigcup_{\delta_\lambda\in K_\lambda}(G_{\delta_\lambda},E)=(F_\lambda,E)\right\}.$$

Then

$$(F,E)=\bigcup_{\lambda\in K}(F_\lambda,E)=\bigcup_{\lambda\in K}\bigcup_{\delta_\lambda\in K_\lambda}(G_{\delta_\lambda},E).$$

For arbitrary $\rho\in\prod\limits_{\lambda\in K}B_\lambda$, since $\bigcup\limits_{\lambda\in K}\bigcup\limits_{(G_{\delta_\lambda},E)\in\rho(\lambda)}(G_{\delta_\lambda},E)=\bigcup\limits_{\lambda\in K}(F_\lambda,E)$,

$$
\begin{aligned}
\tau_\beta(F,E) &= \bigvee_{\underset{\delta\in K}{\cup}(G_\delta,E)=(F,E)}\bigwedge_{\delta\in K}\beta(G_\delta,E)\\
&\geq \bigvee_{\rho\in\prod\limits_{\lambda\in K}B_\lambda}\bigwedge_{\lambda\in K}\bigwedge_{(G_{\delta_\lambda},E)\in\rho(\lambda)}\beta(G_{\delta_\lambda},E)\\
&= \bigwedge_{\lambda\in K}\bigwedge_{\{(G_{\delta_\lambda},E):\delta_\lambda\in K_\lambda\}}\bigwedge_{\delta_\lambda\in K_\lambda}\beta(G_{\delta_\lambda},E)\\
&= \bigwedge_{\lambda\in K}\tau(F_\lambda,E),
\end{aligned}
$$

and

$$\begin{aligned}\tau^*_{\beta^*}(F,E) &= \bigwedge_{\underset{\delta\in K}{\cup}(G_\delta,E)=(F,E)} \bigvee_{\delta\in K} \beta^*(G_\delta,E)\\ &\le \bigwedge_{\rho\in\prod_{\lambda\in K}B_\lambda} \bigvee_{\lambda\in K} \bigvee_{(G_{\delta_\lambda},E)\in\rho(\lambda)} \beta^*(G_{\delta_\lambda},E)\\ &= \bigvee_{\lambda\in K} \bigwedge_{\{(G_{\delta_\lambda},E):\delta_\lambda\in K_\lambda\}} \bigvee_{\delta_\lambda\in K_\lambda} \beta^*(G_{\delta_\lambda},E)\\ &= \bigvee_{\lambda\in K} \tau^*(F_\lambda,E)\end{aligned}$$

are obtained. Thus the pair $\left(\tau_\beta,\tau^*_{\beta^*}\right)$ is an intuitionistic fuzzy topology. It is clear that (β,β^*) is a base of $\left(\tau_\beta,\tau^*_{\beta^*}\right)$. □

Theorem 3.5. *Let (X,E,τ,τ^*) be an $IFTS$ and $Y\subset X$. Define two mappings $\tau_Y,\tau^*_Y : SS(Y,E)\to[0,1]$ by:*

$$\begin{aligned}\tau_Y(F,E) &= \vee\left\{\tau(G,E):(F,E)=(G,E)\widetilde{\cap}\widetilde{Y},\ (G,E)\in SS(X,E)\right\},\\ \tau^*_Y(F,E) &= \wedge\left\{\tau^*(G,E):(F,E)=(G,E)\widetilde{\cap}\widetilde{Y},\ (G,E)\in SS(X,E)\right\}.\end{aligned}$$

Then the pair (τ_Y,τ^*_Y) is an intuitionistic fuzzy topology on Y and

$$\tau_Y\left((G,E)\widetilde{\cap}\widetilde{Y}\right)\ge\tau(G,E),\ \tau^*_Y\left((G,E)\widetilde{\cap}\widetilde{Y}\right)\le\tau(G,E).$$

Proof. For each $(G,E)\in SS(X,E)$ with $(F,E)=(G,E)\widetilde{\cap}\widetilde{Y}$, we have $\tau(G,E)+\tau^*(G,E)\le1$, i.e, $\tau(G,E)\le1-\tau^*(G,E)$. Hence

$$\vee\left\{\tau(G,E):(G,E)\widetilde{\cap}\widetilde{Y}=(F,E)\right\}\le\vee\left\{1-\tau^*(G,E):(G,E)\widetilde{\cap}\widetilde{Y}=(F,E)\right\}$$

$$\begin{aligned}&\Rightarrow\ \vee\left\{\tau(G,E):(G,E)\widetilde{\cap}\widetilde{Y}=(F,E)\right\}\le1-\wedge\left\{\tau^*(G,E):(G,E)\widetilde{\cap}\widetilde{Y}=(F,E)\right\}\\ &\Rightarrow\ \tau_Y(F,E)+\tau^*_Y(F,E)\le1\end{aligned}$$

as required.

$$\begin{aligned}\tau^*_Y(\Phi) &= \wedge\left\{\tau^*(G,E):\tau(G,E)\widetilde{\cap}\widetilde{Y}=\Phi,(G,E)\in SS(X,E)\right\}\\ &\le \tau^*(\Phi)=0.\end{aligned}$$

Therefore, $\tau^*_Y(\Phi)=0$.

$$\begin{aligned}\tau^*_Y(Y) &= \wedge\left\{\tau^*(G,E):\tau(G,E)\widetilde{\cap}\widetilde{Y}=\widetilde{Y},(G,E)\in SS(X,E)\right\}\\ &\le \tau^*\left(\widetilde{X}\right)=0,\end{aligned}$$

so $\tau^*_Y\left(\widetilde{Y}\right)=0$. Similarly, $\tau_Y(\Phi)=\tau_Y\left(\widetilde{Y}\right)=1$ are obtained. Now

$$\tau_Y\left((F_1,E)\widetilde{\cap}(F_2,E)\right)=\vee\left\{\tau(G,E):(G,E)\widetilde{\cap}\widetilde{Y}=(F_1,E)\widetilde{\cap}(F_2,E)\right\}$$

$$\begin{aligned}&\ge\ \vee\left\{\tau\left((G_1,E)\widetilde{\cap}(G_2,E)\right):(G_1,E)\widetilde{\cap}\widetilde{Y}=(F_1,E),(G_2,E)\widetilde{\cap}\widetilde{Y}=(F_2,E)\right\}\\ &=\ \tau_Y(F_1,E)\widetilde{\cap}\tau_Y(F_2,E)\end{aligned}$$

and

$$\tau_Y^*\left((F_1,E)\,\widetilde{\cap}(F_2,E)\right)=\wedge\left\{\tau^*\left(G,E\right):(G,E)\,\widetilde{\cap}\widetilde{Y}=(F_1,E)\,\widetilde{\cap}(F_2,E)\right\}$$

$$\begin{aligned}&\leq \wedge\left\{\tau^*\left((G_1,E)\,\widetilde{\cap}(G_2,E)\right):(G_1,E)\,\widetilde{\cap}\widetilde{Y}=(F_1,E),(G_2,E)\,\widetilde{\cap}\widetilde{Y}=(F_2,E)\right\}\\ &= \tau_Y^*\left(F_1,E\right)\widetilde{\cap}\tau_Y^*\left(F_2,E\right)\end{aligned}$$

hold.

$$\begin{aligned}\tau_Y\left(\underset{i\in\Delta}{\cup}(F_i,E)\right) &= \vee\left\{\tau\left(G,E\right):(G,E)\,\widetilde{\cap}\widetilde{Y}=\underset{i\in\Delta}{\cup}(F_i,E)\right\}\\ &\geq \vee\left\{\tau\left(\underset{i\in\Delta}{\cup}(G_i,E)\right):(G_i,E)\widetilde{\cap}\widetilde{Y}=(F_i,E)\right\}\\ &\geq \vee\left\{\underset{i\in\Delta}{\wedge}\tau(G_i,E):(G_i,E)\widetilde{\cap}\widetilde{Y}=(F_i,E)\right\}\\ &= \underset{i\in\Delta}{\wedge}\left(\vee\left\{\tau(G_i,E):(G_i,E)\widetilde{\cap}\widetilde{Y}=(F_i,E)\right\}\right)\\ &= \underset{i\in\Delta}{\wedge}\tau_Y(F_i,E),\end{aligned}$$

$$\begin{aligned}\tau_Y^*\left(\underset{i\in\Delta}{\cup}(F_i,E)\right) &= \wedge\left\{\tau^*\left(G,E\right):(G,E)\,\widetilde{\cap}\widetilde{Y}=\underset{i\in\Delta}{\cup}(F_i,E)\right\}\\ &\leq \wedge\left\{\tau^*\left(\underset{i\in\Delta}{\cup}(G_i,E)\right):(G_i,E)\widetilde{\cap}\widetilde{Y}=(F_i,E)\right\}\\ &\leq \wedge\left\{\underset{i\in\Delta}{\vee}\tau^*(G_i,E):(G_i,E)\widetilde{\cap}\widetilde{Y}=(F_i,E)\right\}\\ &= \underset{i\in\Delta}{\vee}\left(\wedge\left\{\tau^*(G_i,E):(G_i,E)\widetilde{\cap}\widetilde{Y}=(F_i,E)\right\}\right)\\ &= \underset{i\in\Delta}{\vee}\tau_Y^*(F_i,E).\end{aligned}$$

Hence the pair (τ_Y,τ_Y^*) is an intuitionistic fuzzy topology on Y. It is clear that $\tau_Y\left((G,E)\,\widetilde{\cap}\widetilde{Y}\right)\geq\tau\left(G,E\right)$ and $\tau_Y^*\left((G,E)\,\widetilde{\cap}\widetilde{Y}\right)\leq\tau\left(G,E\right)$ hold. □

Now we define the concept of quotient space of $IFTSs$. Let $\{(X_\lambda,E_\lambda,\tau_\lambda,\tau_\lambda^*)\}_{\lambda\in\wedge}$ be a family of intuitionistic fuzzy topological spaces, different $X_\lambda\cap X_{\lambda'}=\varnothing$ and $E_\lambda\cap E_{\lambda'}=\varnothing$, $\forall\lambda\neq\lambda'$. Let $\widetilde{X}$ be union of all soft points which belong to this space and $E=\underset{\lambda\in\wedge}{\cup}E_\lambda$. Then $\left(\widetilde{X},E\right)$ is a family of soft sets on $X=\underset{\lambda\in\wedge}{\cup}X_\lambda$ with parameters E. For soft point $x_e\in\left(\widetilde{X},E\right)$ if $x\in X_\lambda$, then $e\in E_\lambda$. If $e\in E_\lambda$, then $x\in X_\lambda$. For arbitrary $(F,E)\in\left(\widetilde{X},E\right)$, $(F,E)_\lambda=\{F\left(e\right)\cap X_\lambda\}_{e\in E}$.

Theorem 3.6. *Let $\{(X_\lambda,E_\lambda,\tau_\lambda,\tau_\lambda^*)\}_{\lambda\in\wedge}$ be a family of $IFTSs$, different $X_\lambda's$ be disjoint. Then (τ,τ^*) which is defined by:*

$$\tau\left(F,E\right)=\underset{\lambda\in\wedge}{\wedge}\tau_\lambda\left((F,E)_\lambda\right)\ \textit{and}\ \tau^*\left(F,E\right)=\underset{\lambda\in\wedge}{\vee}\tau_\lambda^*\left((F,E)_\lambda\right),\ \forall\left(F,E\right)\in\left(\widetilde{X},E\right)$$

is an intuitionistic fuzzy topology on X.

Proof. Let (F_1, E), $(F_2, E) \in \left(\widetilde{X}, E\right)$. Then

$$\begin{aligned}
\tau\left((F_1,E)\,\widetilde{\cap}(F_2,E)\right) &= \underset{\lambda\in\wedge}{\wedge}\tau_\lambda\left(\left((F_1,E)\,\widetilde{\cap}(F_2,E)\right)_\lambda\right)\\
&= \underset{\lambda\in\wedge}{\wedge}\tau_\lambda\left((F_1,E)_\lambda\,\widetilde{\cap}(F_2,E)_\lambda\right)\\
&\geq \underset{\lambda\in\wedge}{\wedge}\left(\tau_\lambda\left((F_1,E)_\lambda\right)\wedge\tau_\lambda\left((F_2,E)_\lambda\right)\right)\\
&= \left(\underset{\lambda\in\wedge}{\wedge}\tau_\lambda\left((F_1,E)_\lambda\right)\right)\wedge\left(\underset{\lambda\in\wedge}{\wedge}\tau_\lambda\left((F_2,E)_\lambda\right)\right)\\
&= \tau\left(F_1,E\right)\wedge\tau\left(F_2,E\right)
\end{aligned}$$

and

$$\begin{aligned}
\tau^*\left((F_1,E)\,\widetilde{\cap}(F_2,E)\right) &= \underset{\lambda\in\wedge}{\vee}\tau^*_\lambda\left(\left((F_1,E)\,\widetilde{\cap}(F_2,E)\right)_\lambda\right)\\
&= \underset{\lambda\in\wedge}{\vee}\tau^*_\lambda\left((F_1,E)_\lambda\,\widetilde{\cap}(F_2,E)_\lambda\right)\\
&\leq \underset{\lambda\in\wedge}{\vee}\left(\tau^*_\lambda\left(F_1,E\right)_\lambda\vee\tau^*_\lambda\left(F_2,E\right)_\lambda\right)\\
&= \left(\underset{\lambda\in\wedge}{\vee}\tau^*_\lambda\left((F_1,E)_\lambda\right)\right)\vee\left(\underset{\lambda\in\wedge}{\vee}\tau^*_\lambda\left((F_2,E)_\lambda\right)\right)\\
&= \tau^*\left(F_1,E\right)\vee\tau^*\left(F_2,E\right)
\end{aligned}$$

are satisfied.

Secondly, let $\{(F_i, E_i)\}_{i\in I}$ be a family of soft sets.

$$\begin{aligned}
\tau\left(\underset{i\in I}{\cup}(F_i,E_i)\right) &= \underset{\lambda\in\wedge}{\wedge}\tau_\lambda\left(\left(\underset{i\in I}{\cup}(F_i,E_i)\right)_\lambda\right)\\
&= \underset{\lambda\in\wedge}{\wedge}\tau_\lambda\left(\underset{i\in I}{\cup}(F_i,E_i)_\lambda\right)\\
&\geq \underset{\lambda\in\wedge}{\wedge}\underset{i\in I}{\wedge}\tau_\lambda\left((F_i,E_i)_\lambda\right)\\
&= \underset{i\in I}{\wedge}\left(\underset{\lambda\in\wedge}{\wedge}\tau_\lambda\left((F_i,E_i)_\lambda\right)\right) = \underset{i\in I}{\wedge}\tau\left((F_i,E_i)\right),
\end{aligned}$$

$$\begin{aligned}
\tau^*\left(\underset{i\in I}{\cup}(F_i,E_i)\right) &= \underset{\lambda\in\wedge}{\vee}\tau^*_\lambda\left(\left(\underset{i\in I}{\cup}(F_i,E_i)\right)_\lambda\right)\\
&= \underset{\lambda\in\wedge}{\vee}\tau^*_\lambda\left(\underset{i\in I}{\cup}(F_i,E_i)_\lambda\right)\\
&\leq \underset{\lambda\in\wedge}{\vee}\underset{i\in I}{\vee}\tau^*_\lambda\left((F_i,E_i)_\lambda\right)\\
&= \underset{i\in I}{\vee}\left(\underset{\lambda\in\wedge}{\vee}\tau^*_\lambda\left((F_i,E_i)_\lambda\right)\right) = \underset{i\in I}{\vee}\tau^*\left((F_i,E_i)\right)
\end{aligned}$$

are obtained. Thus (X, E, τ, τ^*) is an $IFTS$. □

4. Conclusion

Soft topological spaces are important generalization of topological spaces. In this paper, we give the definition of intuitionistic fuzzy topology (cotopology) (τ, τ^*) which is a mapping satisfying some definite conditions from $SS(X, E)$ to $[0, 1]$. It is shown that an intuitionistic fuzzy topological space gives a parameterized family of soft bitopologies on X. We introduce the concepts of base and subbase of intuitionistic fuzzy topological spaces on soft sets. We hope that the results of this study may help in the investigation of intuitionistic fuzzy topological spaces on soft sets.

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