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EXPLORING NEW FEATURES FOR THE PERTURBED CHEN-LEE-LIU MODEL VIA $(m+\frac{1}{G'})$-EXPANSION METHOD

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Abstract. In this work, the perturbed Chen-Lee-Liu equation, which describes the propagation of an optical pulse in plasma and optical fiber is studied. The $(m+\frac{1}{G'})$-expansion method is used for this purpose. As a result, bright-singular, dark-singular, dark and periodic optical soliton waves are constructed. Specific values for the parameters under conditions are also provided to display the pulse propagation of the found solutions.

1. Introduction

Nonlinear partial differential equations are used to investigate the properties of several physics models. The Schrödinger equations are one kind of these equations. Because such equations play an important role in areas such as mathematic physics, optic, plasma, and fiber-optic telecommunications engineering, it is essential to evaluate and study their wave solutions.

Exact solutions to nonlinear Schrödinger's equation are essential in applied mathematics. Several approaches for obtaining exact solutions to nonlinear partial differential equations have been proposed, including the simplified Hirota method [23, 12], the modified Kudryashov method [9, 1], the extended sinh-Gordon expansion method [5, 20], an extended F-expansion method [21], the symbolic computational method [11, 6], the Jacobi elliptic function method [4, 14], the generalized exponential rational function method [13], the sine–Gordon expansion method [2], the Bernoulli sub-ODE method [3] and so on.

In this manuscript, we consider the perturbed Chen-Lee-Liu (CLL) model [24]

$$i\psi_t+\alpha\psi_{xx}+i\beta|\psi|^2\psi_x=i[\gamma\psi_x+\mu(|\psi|^{2n}\psi)_x+\delta(|\psi|^{2n})_x\psi], \tag{1.1}$$

here γ is the inter-modal dispersion coefficient, μ is a coefficient of self-steepening for short pulses and δ characterizes the coefficient of nonlinear dispersion. Also, α represents a coefficient of the group velocity dispersion, and finally β symbolizes a coefficient of nonlinearity.

In this study, we investigate Eq. (1.1) at positive integers with $n=1$

$$i\psi_t+\alpha\psi_{xx}+i\beta|\psi|^2\psi_x=i[\gamma\psi_x+\mu(|\psi|^2\psi)_x+\delta(|\psi|^2)_x\psi]. \tag{1.2}$$

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We established solutions with the help of the Jacobi elliptic functions method to the perturbed Chen-Lee-Liu equation that represents the propagation of an optical pulse in plasma and optical fiber. Recently, some authors tried to apply some methods to investigate the CLL model such as the modified extended tanh expansion method was used to find solitary wave solutions [24]. Yokus et al. used the modified Kudryashov and $(\frac{1}{G'})$-expansion methods [8]. In [15], the Riccati method has been employed. Zhang et al. [10] have investigated qualitative analysis and the bifurcation method. Optical solutions of the studied equation via the trial equation approach have been constructed [22]. Apart from these, many studies have been made and continue to be done for the Chen-Lee-Liu equation [7, 19, 17]. Akbar and others studied the CLL model via using different solutions functions with help of the Jacobi elliptic functions [16]. Kudryashov found general solutions by using different methods with the elliptic function approach [18].

In this article, we use $(m+\frac{1}{G'})$-expansion method to reveal some new novel solutions for the perturbed Chen-Lee-Liu equation that characterize the propagation of an optical pulse in plasma and optical fiber.

This research is arranged as follows, with an introduction in Section 1. In Section 2, we focused on introducing the $(m+\frac{1}{G'})$-expansion method. In Section 3, we used the provided approach to investigate the new precise solutions to the perturbed Chen-Lee-Liu model. Section 4 presents the study's result.

2. The $(m+\frac{1}{G'})$-expansion method

Consider the following PDE in two variables in order to describe the $\left(m+{}^{1}/_{G'}\right)$-expansion method as follows:

$$O\left(\psi,\psi_x,\psi_t,\psi_{xt},\quad \ldots\right)=0. \tag{2.1}$$

Here O is a polynomial function of its inputs in general, and the subscripts signify partial derivatives. The key steps of the $\left(m+{}^{1}/_{G'}\right)$-expansion approach will now be described.

Step 1: Assume that equation (2.1) has a traveling solution of the following form:

$$\psi\left(x,t\right)=u(\xi)e^{i\theta(x,t)},\quad \xi=x-\rho t\quad \theta\left(x,t\right)=-kx+wt+\eta. \tag{2.2}$$

Substituting Eq. (2.2) into Eq. (2.1), a result is a nonlinear ordinary differential equation (NLODE) as follows:

$$P\left(u,u',u'',\quad \ldots\right)=0, \tag{2.3}$$

where P is a polynomial of $u(\xi)$ and u', u'', $\cdots$ are total derivatives, as well as the prime $'$ signifies $\frac{d}{d\zeta}$.

Step 2: We assume that equation (2.3) has the following form:

$$\begin{aligned} u\left(\xi\right)=\sum_{i=-n}^{n}a_i(m+F)^i&=a_{-n}(m+F)^{-n}+\cdots+a_{-1}(m+F)^{-1}\\ &+m\,a_0+a_1\left(m+F\right)+\cdots+a_n(m+F)^n, \end{aligned} \tag{2.4}$$

where a_n $(n=0,\pm1,\ldots,\pm n)$ and m are nonzero constants, which will be evaluated later. The value of n will be evaluated according to the principles of balance and

$$Q = \frac{1}{G'} \text{ where } G(\xi) \text{ verify } G'' + (\lambda + 2m\mu)\, G' + \mu = 0. \tag{2.5}$$

Step 3: Putting Eq. (2.4) into Eq. (2.3) then collecting all terms that have the same order of the $\left(m + {}^{1}/_{G'}\right)^{n}$, then making these terms equal to zero, gives us a set of algebraic equations, which can be solved by using the computational software to evaluate the values ofa_n, $n = 0, 1, \dots, n$, α and w. Step 4: Inserting the obtained values into Eq. (2.4) gives us solutions of the Eq. (2.1).

3. Application to the $(m + \frac{1}{G'})$-expansion method

In this section, we apply the $m + \frac{1}{G'}$-expansion method to Eq. (1.2). Firstly, by inserting Eq. (2.2) into Eq. (1.2), we get

$$\begin{aligned} &- i\rho u' - wu + \alpha u'' - 2k\alpha i u' - \alpha k^2 U + i\beta u^2 u' \\ &+ \beta k u^3 - i\gamma u' - \gamma k u - 3i\mu u^2 u' - \mu k u^3 - 2i\delta u^2 u' = 0. \end{aligned} \tag{3.1}$$

by expressing the following parts of Eq.(3.1) as follows: The reel part becomes

$$(-w - \alpha k^2 - \gamma k)u + \alpha u'' + k(\beta - \mu)u^3 = 0, \tag{3.2}$$

and the imaginary part is

$$(-\rho - 2k\alpha - \gamma)u' + (\beta - 3\mu - 2\delta)u^2 u' = 0. \tag{3.3}$$

Then, by setting the coefficients of the components of the imaginary part equal to zero, we obtain $\rho = -2k\alpha - \gamma$ and $\beta = 3\mu + 2\delta$. Plugging these values into the real part yields

$$(-w - \alpha k^2 - \gamma k)u + \alpha u'' + 2k(\beta + \mu)u^3 = 0. \tag{3.4}$$

By utilizing the balance principle, one can get $n = 1$. Plugging Eq. (2.4), one can get the solution of Eq. (3.4) like:

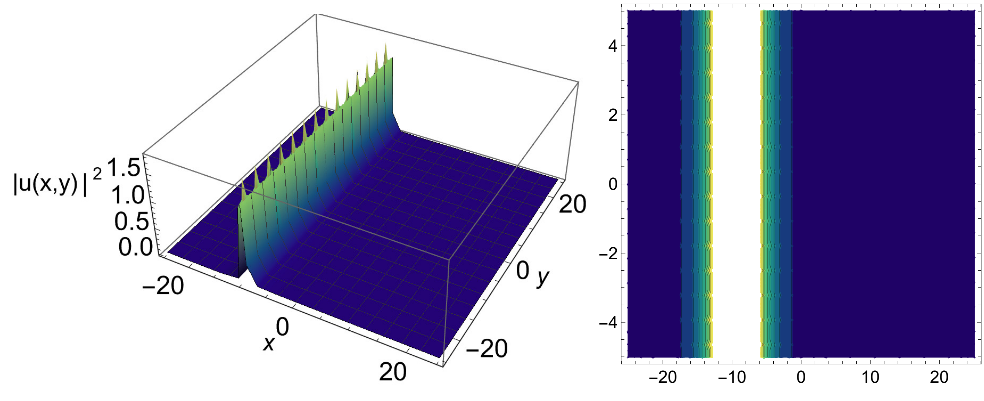
$$u(\xi) = a_{-1}(m + F)^{-1} + m\, a_0 + a_1 (m + F)\,. \tag{3.5}$$

when we substituting Eq. (3.5) into Eq. (3.4), we find the solutions by taking into account the equation system obtained for the following conditions by performing the necessary operations.

Case 1. When we have $a_{-1} = 0, a_0 = \frac{\sqrt{m}\sqrt{\alpha}\lambda}{2\sqrt{k(-m\delta+\lambda)}}, a_1 = -\frac{\sqrt{\alpha}\lambda}{\sqrt{m}\sqrt{k(-m\delta+\lambda)}}, w = -k(k\alpha + \gamma) - \frac{\alpha\lambda^2}{2}, \mu = -\frac{\lambda}{m}$, and putting this case into Eq. (3.5) then into Eq. (2.2) we get

$$\psi(x,t) = -\frac{e^{-\frac{1}{2}i\left(2k(x+t(k\alpha+\gamma))-2\eta+t\alpha\lambda^2\right)}\sqrt{m\alpha}\lambda\left(1 + A_1 e^{(x+t(2k\alpha+\gamma))\lambda} m\right)}{2\left(mA_1 e^{(x+t(2k\alpha+\gamma))\lambda} - 1\right)\sqrt{k(\lambda - m\delta)}}. \tag{3.6}$$

This is bright-singular optical solution as presented in Fig. 1.



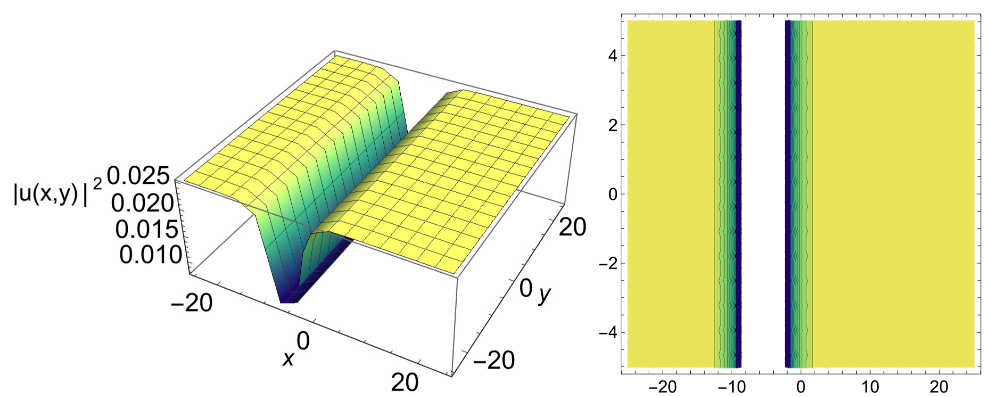
(A) 3D figure (B) Corresponding contour plot

FIGURE 1. Bright-singular solution drawn when $\lambda = 1, m = 1, \eta = 1, k = 5, A1 = 3, a_0 = 2, \delta = -1/2, \lambda] = 1/2, \alpha = 1/3, \gamma = 1/4, t = 2$.

Case 2: When $a_{-1} = -\frac{m\sqrt{\alpha}(\lambda+m\mu)}{\sqrt{-k(\delta+\mu)}}$, $a_0 = \frac{\sqrt{\alpha}\lambda}{2\sqrt{-k(\delta+\mu)}}$, $a_1 = 0$, $w = -k^2\alpha - k\gamma - \frac{1}{2}\alpha(\lambda+2m\mu)^2$ and plugging this case into Eq. (3.5) then into Eq. (2.2) we obtain

$$\psi(x,t) = \frac{\sqrt{\alpha}\,(\lambda+2m\mu)\left(\mathrm{e}^{(x+t(2k\alpha+\gamma))(\lambda+2m\mu)}\,(\lambda+m\mu) - A_1 m\,(\lambda+2m\mu)\right)}{2\sqrt{-k\,(\delta+\mu)}\left(\mathrm{e}^{(x+t(2k\alpha+\gamma))(\lambda+2m\mu)}\,(\lambda+m\mu) + A_1 m\,(\lambda+2m\mu)\right)} \mathrm{e}^{-\frac{1}{2}\mathrm{i}\left(2k^2t\alpha+2k(x+t\gamma)-2\eta+t\alpha(\lambda+2m\mu)^2\right)}. \tag{3.7}$$

This is dark-singular optical solution as presented in Fig. 2.



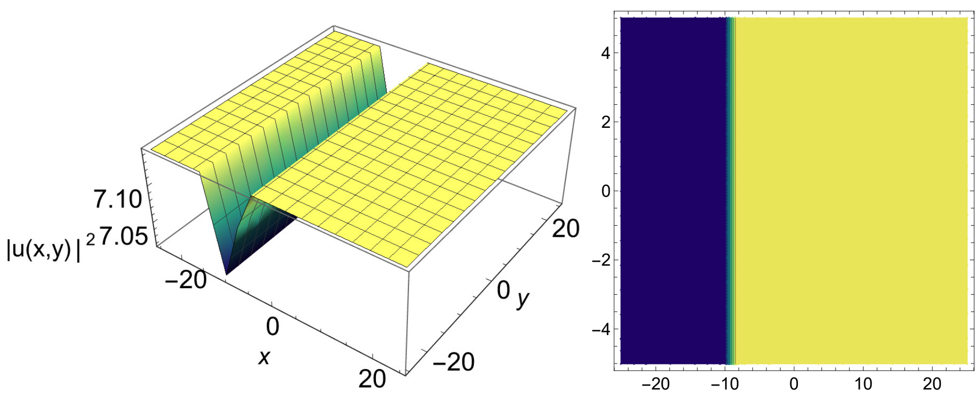
(A) 3D figure (B) Corresponding contour plot

FIGURE 2. dark-Singular solution plotted when $\lambda = 1, m = 1, \eta = 1, k = 5, A_1 = 3, a_0 = 2, \delta = -1/2, \lambda = 1/2, \alpha = 1/3, \gamma = 1/4, t = 2, \mu = 1/8$.

Case 3: When $a_{-1} = 0$, $a_0 = -\frac{\sqrt{w+k(k\alpha+\gamma)}}{\sqrt{2}\sqrt{k(\delta+\mu)}}$, $a_1 = \frac{\sqrt{\alpha(-w-k(k\alpha+\gamma))}\mu}{\sqrt{w+k(k\alpha+\gamma)}\sqrt{k(\delta+\mu)}}$, $\lambda = \frac{\sqrt{2(-w-k(k\alpha+\gamma))}}{\sqrt{\alpha}}$, $m = -\frac{\sqrt{2(-w-k(k\alpha+\gamma))}}{\sqrt{\alpha}\mu}$ and inserting this case into Eq. (3.5) then into Eq. (2.2) we obtain

$$\psi(x,t) = \frac{\left(A_1 e^{\frac{\sqrt{2(-w-k(k\alpha+\gamma))}(x+t(2k\alpha+\gamma))}{\sqrt{\alpha}}}\sqrt{2\left(-w-k\left(k\alpha+\gamma\right)\right)} - \sqrt{\alpha}\mu\right)}{\sqrt{k\left(\delta+\mu\right)}\left(2A_1 e^{\frac{\sqrt{2(-w-k(k\alpha+\gamma))}(x+t(2k\alpha+\gamma))}{\sqrt{\alpha}}}\sqrt{-w-k\left(k\alpha+\gamma\right)} + \sqrt{2\alpha}\mu\right)} e^{i(tw-kx+\eta)}\sqrt{w+k\left(k\alpha+\gamma\right)}. \tag{3.8}$$

This is dark optical solution as presented in Fig. 3.



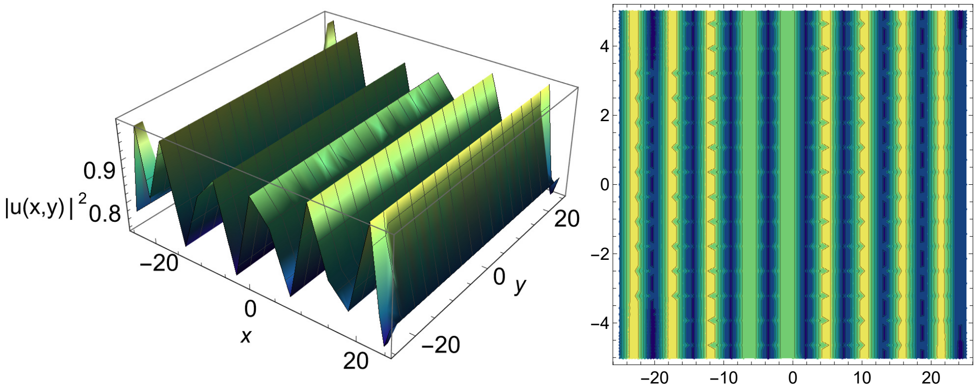
(A) 3D figure (B) Corresponding contour plot

FIGURE 3. Dark solution plotted when $\lambda = 1, m = 1, \eta = 1, k = -1/5, A_1 = 3, a_0 = 2, \delta = 1/2, \lambda = 1/2, \alpha = 1/3, \gamma = 4, t = 2, \mu = 1/8, w = -1$.

Case 4: When $m = -\frac{\sqrt{-2\alpha(w+k(k\alpha+\gamma))\mu^2}+2\mu\sqrt{\alpha(-k(\delta+\mu))}a_0}{2\alpha\mu^2}, a_{-1} = 0$, $a_1 = \frac{\sqrt{\alpha}\mu}{\sqrt{-k(\delta+\mu)}}$, $\lambda = \frac{2\sqrt{-k(\delta+\mu)}a_0}{\sqrt{\alpha}}$, and substituting this case into Eq. (3.5) then into Eq. (2.2) we obtain

$$\psi(x,t) = \frac{e^{i(tw-kx+\eta)}\sqrt{\alpha}\mu\left(2A_1 e^{\frac{\sqrt{2}(x+t(2k\alpha+\gamma))\sqrt{\rho}}{\alpha\mu}}\left(w+k\left(k\alpha+\gamma\right)\right)+\sqrt{2\rho}\right)}{2\sqrt{-k\left(\delta+\mu\right)}\left(\alpha\mu^2+\sqrt{2}A_1 e^{\frac{\sqrt{2}(x+t(2k\alpha+\gamma))\sqrt{\rho}}{\alpha\mu}}\sqrt{\rho}\right)}, \tag{3.9}$$

where $\rho = -\alpha\left(w+k\left(k\alpha+\gamma\right)\right)\mu^2$. As shown in Fig. 4, Eq. (3.9) is a periodic optical solution.



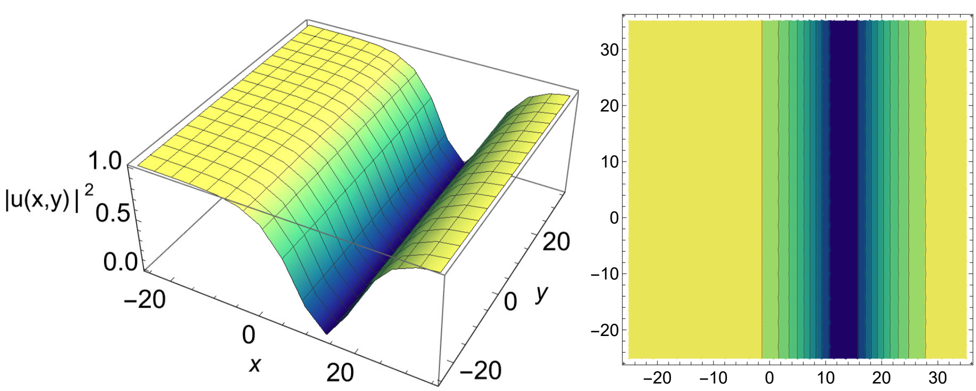
(A) 3D figure (B) Corresponding contour plot

FIGURE 4. Periodic solution plotted when $\lambda = 1, m = 1, \eta = 1, k = -1/5, A_1 = 3, a_0 = 2, \delta = 1/2, \lambda = 1/2, \alpha = 1/3, \gamma = 4, t = 2, \mu = 1/8, w = 1$.

Case 5: When $a_{-1} = -\frac{2m(\lambda+m\mu)a_0}{\lambda}, a_1 = 0$, $\gamma = -\frac{w}{k} + \frac{2(\delta+\mu)\left(2k^2+(\lambda+2m\mu)^2\right)a_0^2}{\lambda^2}$, $\alpha = -\frac{4k(\delta+\mu)a_0^2}{\lambda^2}$, and taking into account this case into Eq. (3.5) then into Eq. (2.2) we have

$$\psi(x,t) = a_0 e^{i(tw-kx+\eta)} - \frac{2a_0 m\left(\lambda+m\mu\right)e^{i(tw-kx+\eta)}}{\lambda\left(m + \frac{1}{A_1 e^{(\lambda+2m\mu)\left(\frac{tw}{k}-x-\frac{2t(\delta+\mu)\left((\lambda+2m\mu)^2-2k^2\right)a_0^2}{\lambda^2}\right)} - \frac{\mu}{\lambda+2m\mu}}\right)}. \tag{3.10}$$

As shown in Fig. 5, Eq. (3.10) is a dark optical solution.



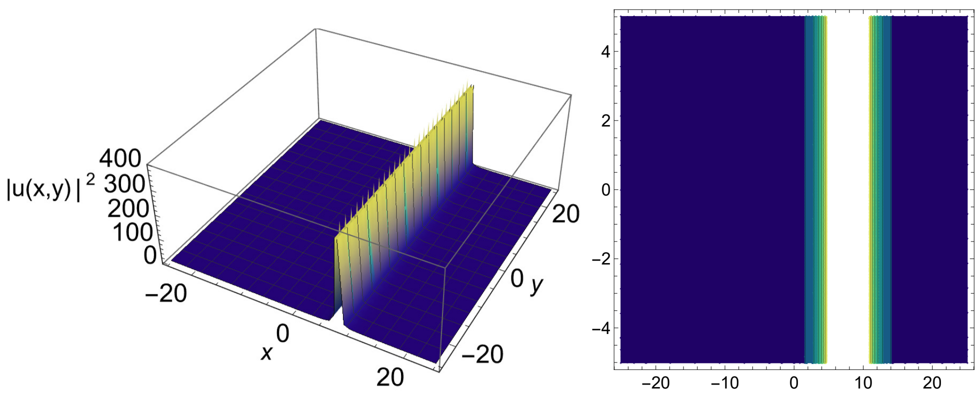
(A) 3D figure (B) Corresponding contour plot

FIGURE 5. Dark solution plotted when $\lambda = 1, m = 1, \eta = 1, k = 1/5, A_1 = 3, a_0 = 2, \delta = 1/2, \lambda = 1/2, \alpha = 1/3, \gamma = 4, t = 2, \mu = -1/8, w = 1$.

Case 6: When $a_{-1} = 0, a_1 = \frac{2\mu a_0}{\lambda}, \gamma = -\frac{w}{k} + \frac{2(\delta+\mu)\left(2k^2+(\lambda+2m\mu)^2\right)a_0^2}{\lambda^2}, \alpha = -\frac{4k(\delta+\mu)a_0^2}{\lambda^2}$, and putting this case into Eq. (3.5) then into Eq. (2.2) we have

$$\psi(x,t) = a_0 e^{i(tw-kx+\eta)} + \frac{2m\mu a_0}{\lambda} e^{i(tw-kx+\eta)} + \frac{2\mu a_0 e^{i(tw-kx+\eta)}}{\lambda A_1 e^{(\lambda+2m\mu)\left(\frac{tw}{k} - x - \frac{2t(\delta+\mu)\left((\lambda+2m\mu)^2-2k^2\right)a_0^2}{\lambda^2}\right)} - \frac{\lambda\mu}{\lambda+2m\mu}}. \tag{3.11}$$

Eq. (3.11) is a bright-singular optical solution as seen in fig. 6.



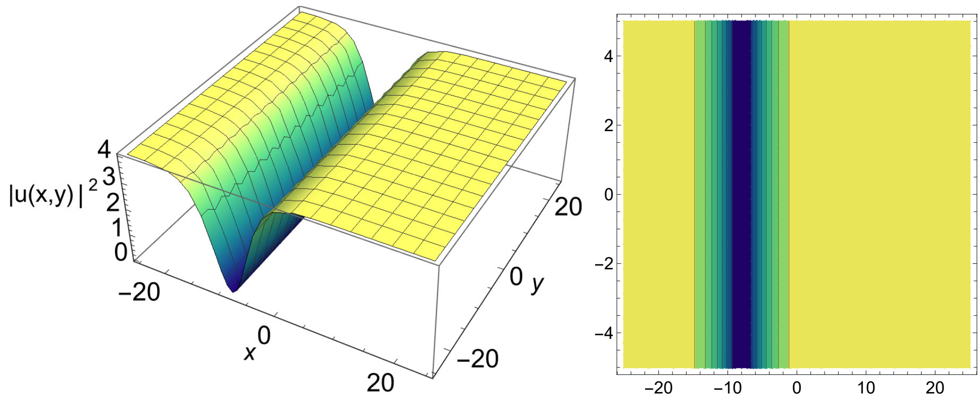
(A) 3D figure (B) Corresponding contour plot

Figure 6. bright-singular solution plotted when $\lambda = 1, m = 1, \eta = 1, k = 1/5, A_1 = 3, a_0 = 1, \delta = 1/2, \lambda = 1/2, \alpha = 1/3, \gamma = 4, t = 2, \mu = -1/2, w = 1$.

Case 7: When $a_{-1} = 0$, $a_1 = -\frac{2a_0}{m}$, $w = -k\left(k\alpha+\gamma\right) - \frac{\alpha\lambda^2}{2}$, $\mu = -\frac{\lambda}{m}$, $\delta = \frac{\lambda}{m} - \frac{\alpha\lambda^2}{4ka_0^2}$, and inserting this case into Eq. (3.5) then into Eq. (2.2) yields

$$\psi(x,t) = -\frac{e^{-\frac{1}{2}i\left(2k(x+t(k\alpha+\gamma))-2\eta+t\alpha\lambda^2\right)}\left(1+A_1 e^{(x+t(2k\alpha+\gamma))\lambda} m\right)a_0}{A_1 m e^{(x+t(2k\alpha+\gamma))\lambda}-1}. \tag{3.12}$$

Eq. (3.12) is a dark optical solution as seen in fig. 7.



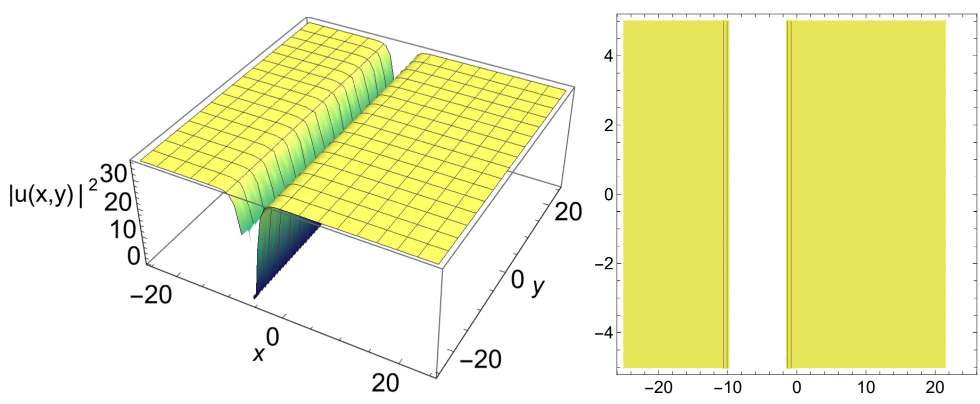
(A) 3D figure (B) Corresponding contour plot

FIGURE 7. Dark solution plotted when $\lambda = 1, m = -1/2, \eta = 1, k = 5, A_1 = 3, a_0 = 2, \delta = -1/2, \lambda = 1/2, \alpha = 1/3, \gamma = 1/4, t = 2$.

Case 8: when $a_{-1} = -\frac{2m(\lambda+m\mu)a_0}{\lambda}, a_1 = 0,\ w = -k^2\alpha - k\gamma - \frac{1}{2}\alpha(\lambda+2m\mu)^2$, $\delta = -\mu - \frac{\alpha\lambda^2}{4ka_0^2}$ and taking into account this case into Eq. (3.5) then into Eq. (2.2) we can get

$$\psi(x,t) = \left(a_0 - \frac{2m\,(\lambda+m\mu)\,a_0}{\lambda\left(m + \frac{1}{A_1 e^{-((x+2kt\alpha+t\gamma)(\lambda+2m\mu)) - \frac{\mu}{\lambda+2m\mu}}}\right)} \right) e^{-\frac{1}{2}i\left(2k^2t\alpha+2k(x+t\gamma)-2\eta+t\alpha(\lambda+2m\mu)^2\right)}. \tag{3.13}$$

Eq. (3.12) is a dark-singular optical solution as seen in fig. 8.



(A) 3D figure (B) Corresponding contour plot

FIGURE 8. Dark-singular solution plotted when $\lambda = 1, m = 1, \eta = , k = 5, A_1 = 3, a_0 = -2, \delta = 2, \lambda = -1/2, \alpha = 1/3, \gamma = 1/4, t = 2, \mu = 1$.

4. Conclusions

In this article, we have found several novel solutions to the perturbed Chen-Lee-Liu equation by using the $(m + \frac{1}{G'})$-expansion method. These solutions are bright-singular, dark-singular, dark and periodic optical waves. These solutions characterize the plasma propagation experienced with electron density in the electromagnetic environment. The existence of gained solutions is verified and constraint conditions are utilized.

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